The Manipulability of Centrality Measures
An Axiomatic Approach

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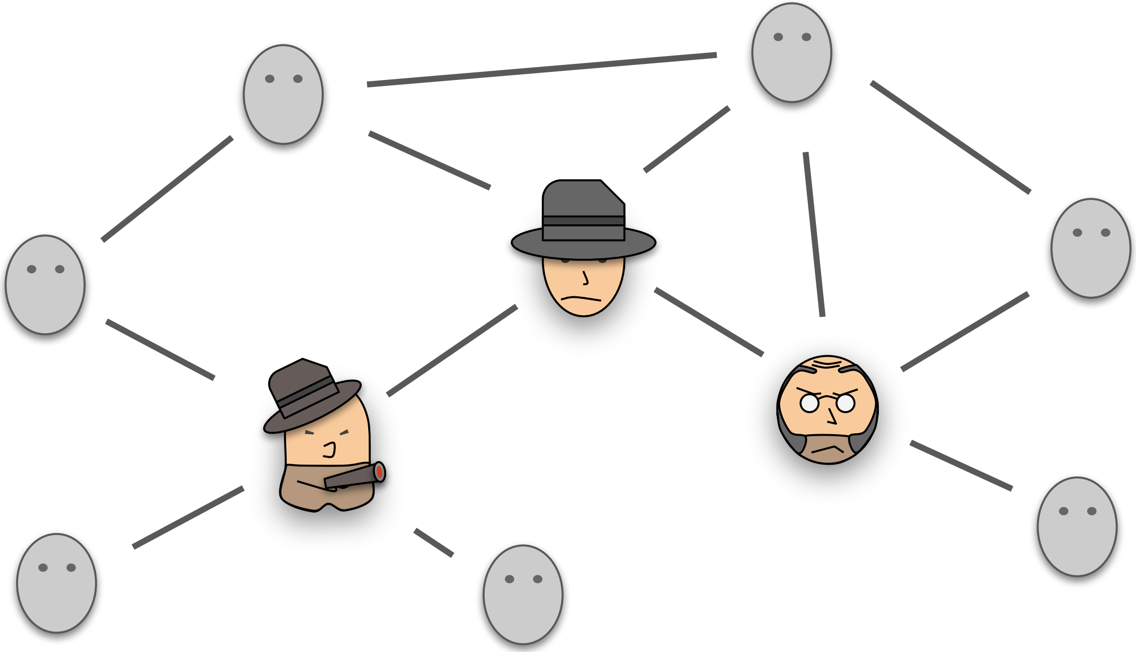
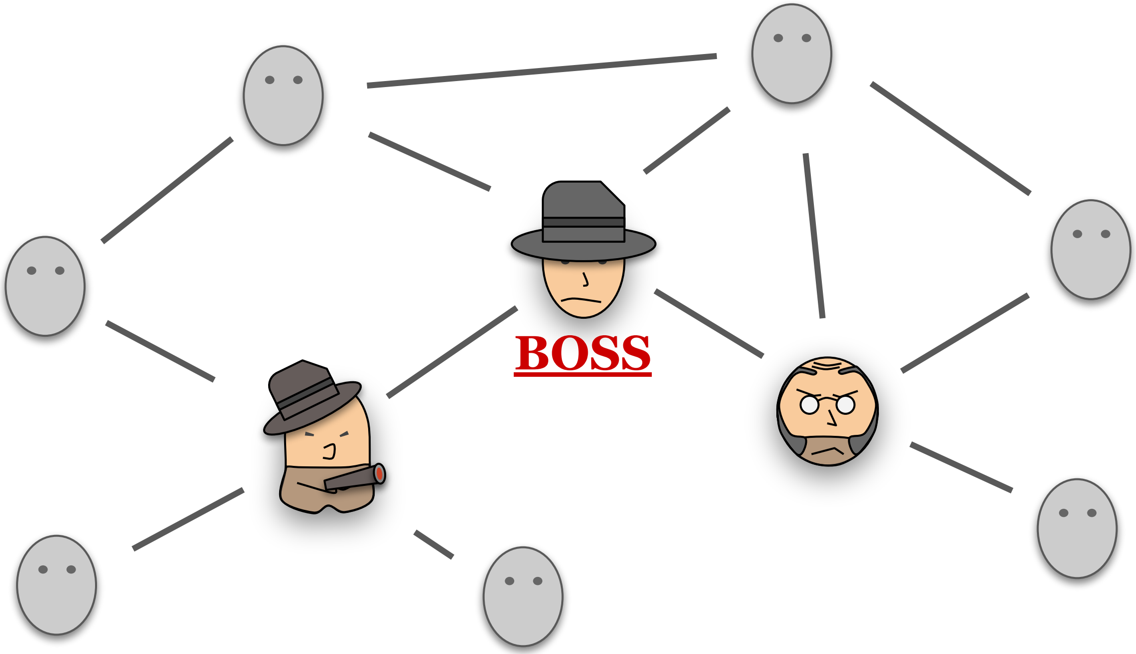
Motivation

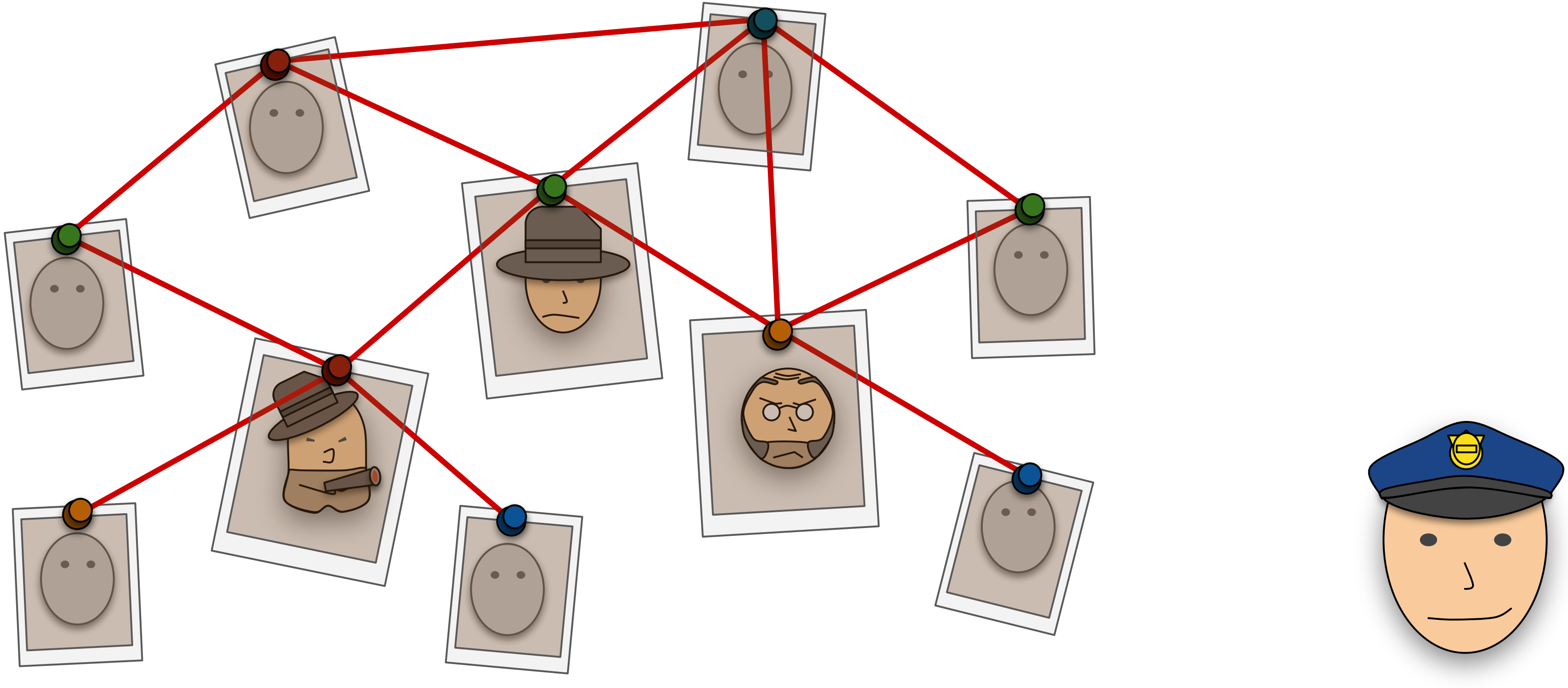
Investigation of criminal networks



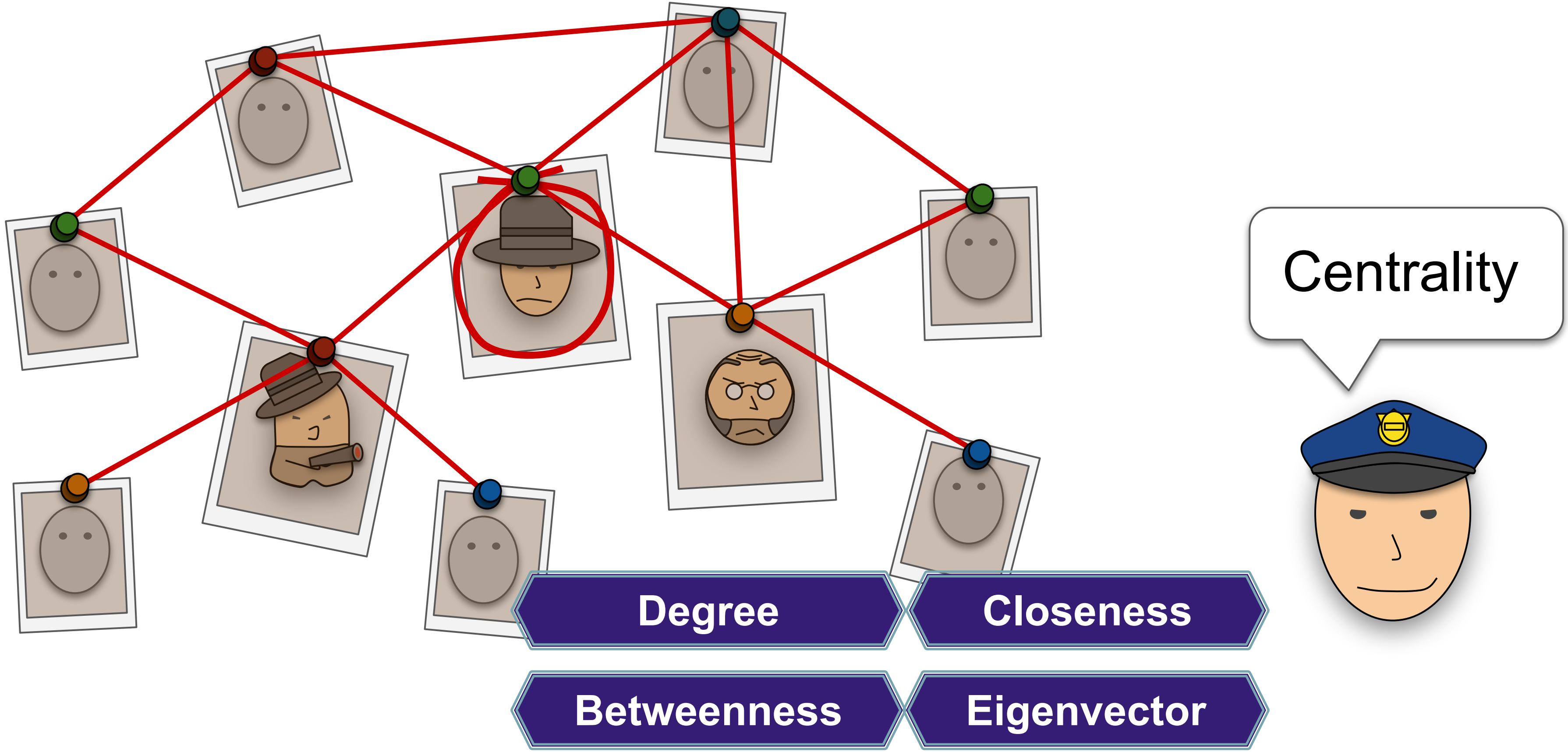
Investigation of criminal networks



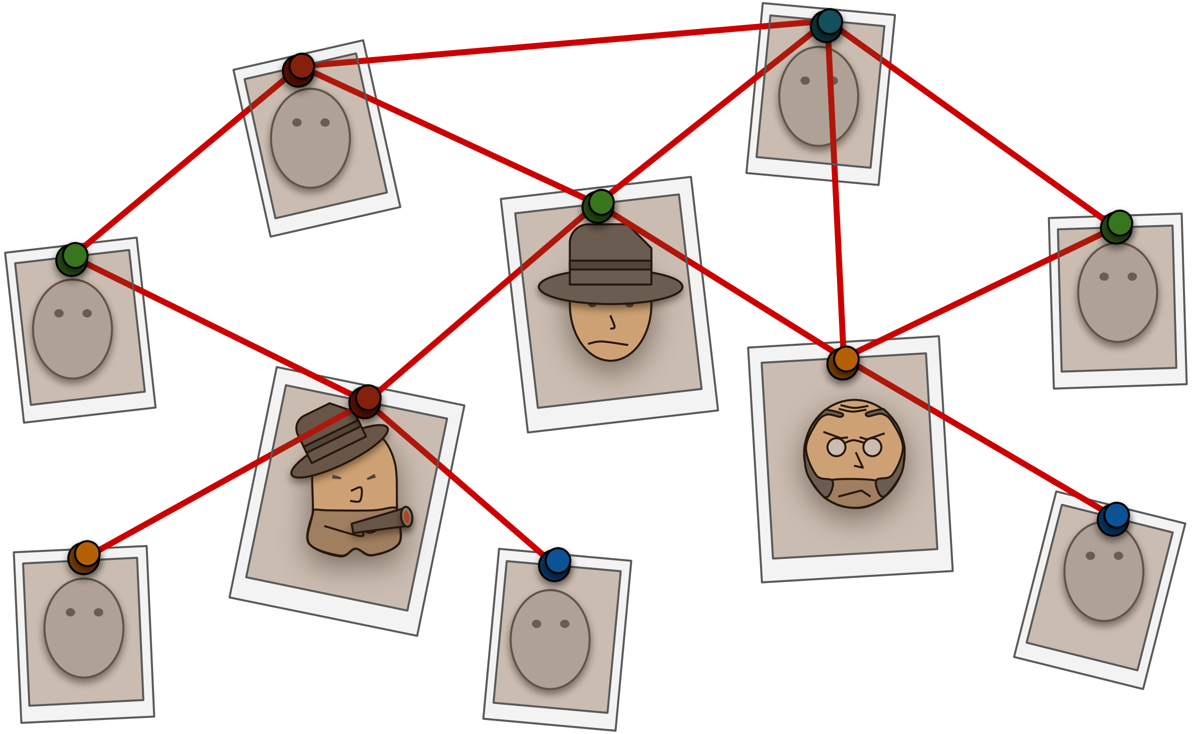
Investigation of criminal networks



Investigation of criminal networks

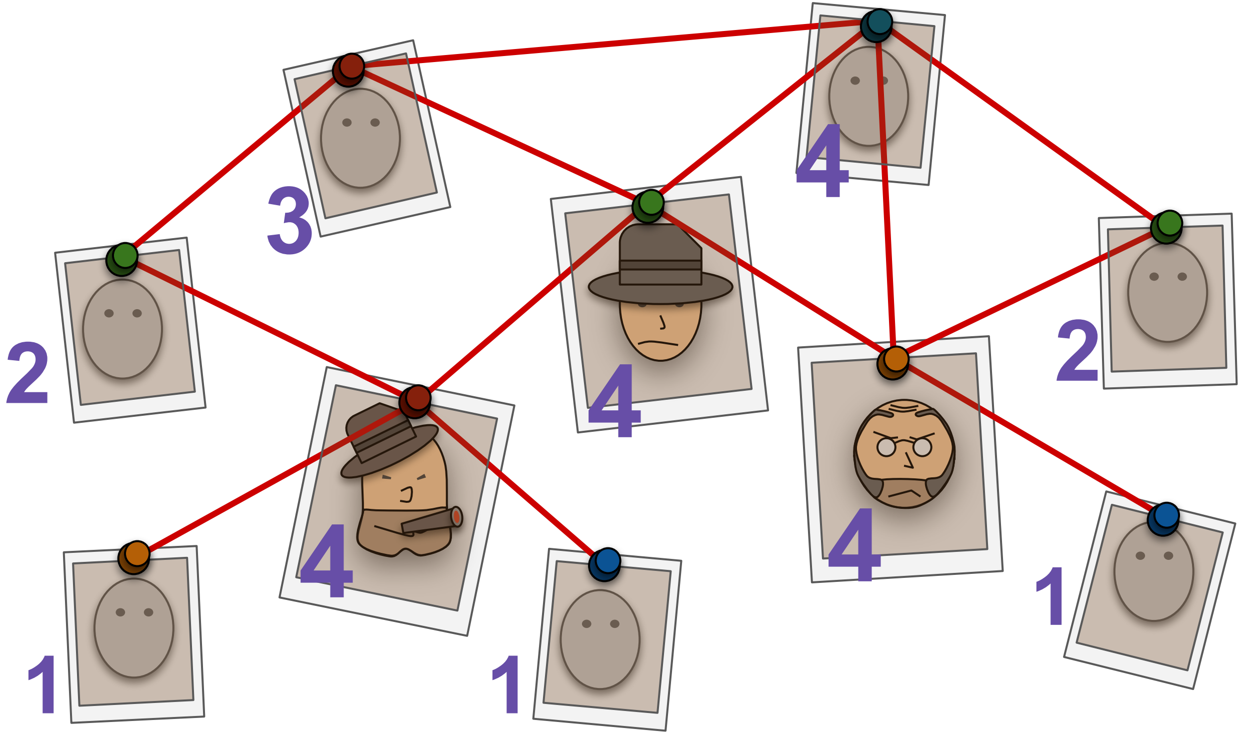


Digression - Centrality measures



Functions assigning value to nodes reflecting their importance

Digression - Centrality measures

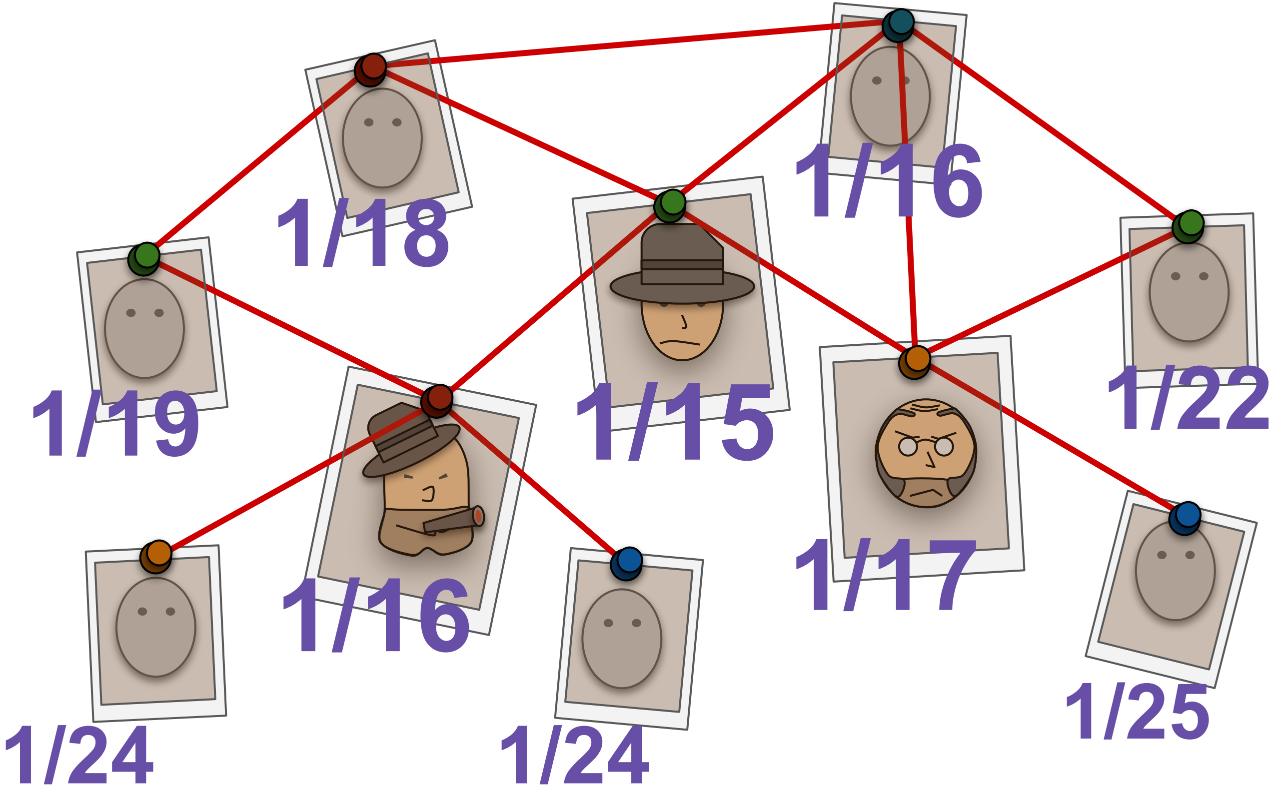


Functions assigning value to nodes reflecting their importance

Degree

The number of connections

Digression - Centrality measures



Functions assigning value to nodes reflecting their importance

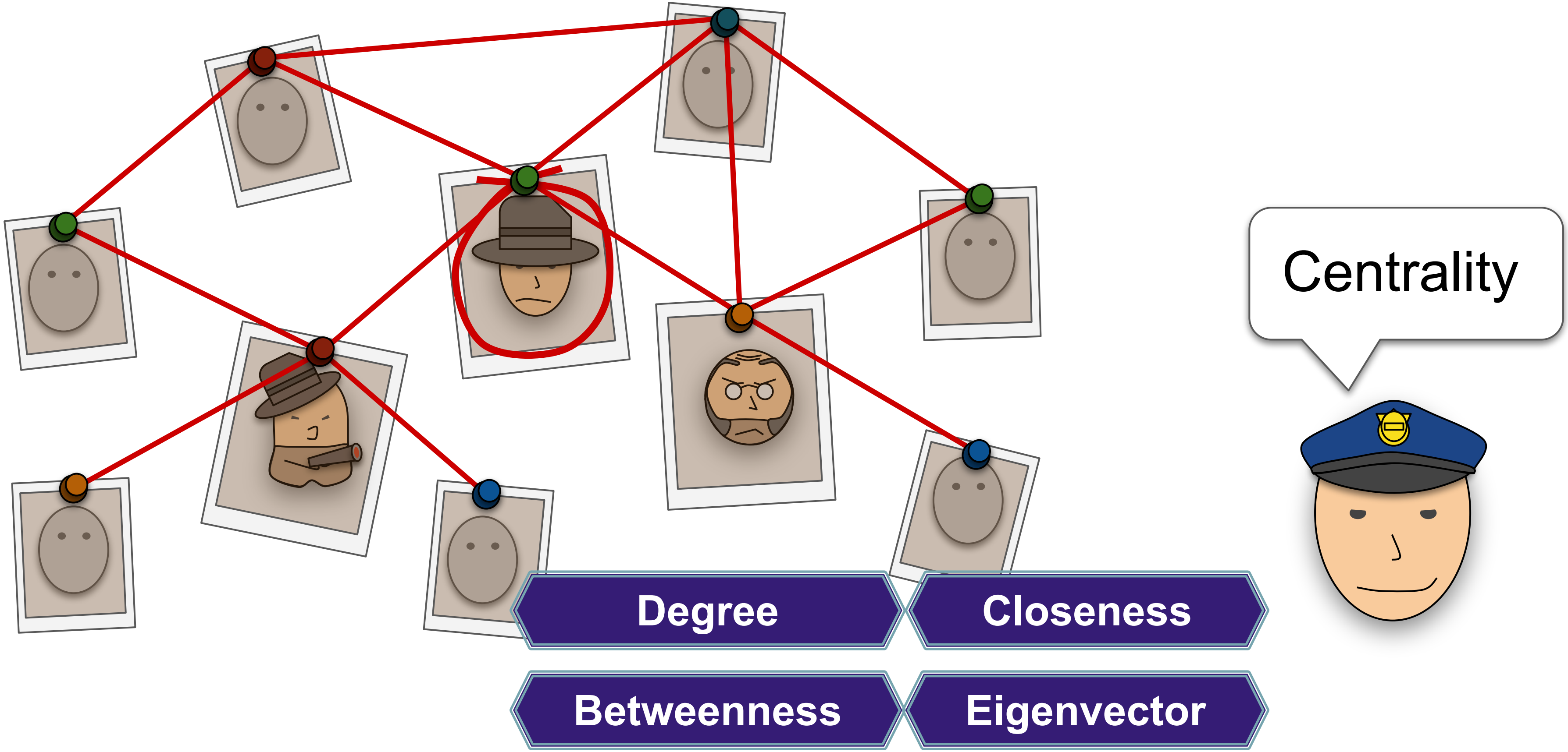
Degree

The number of connections

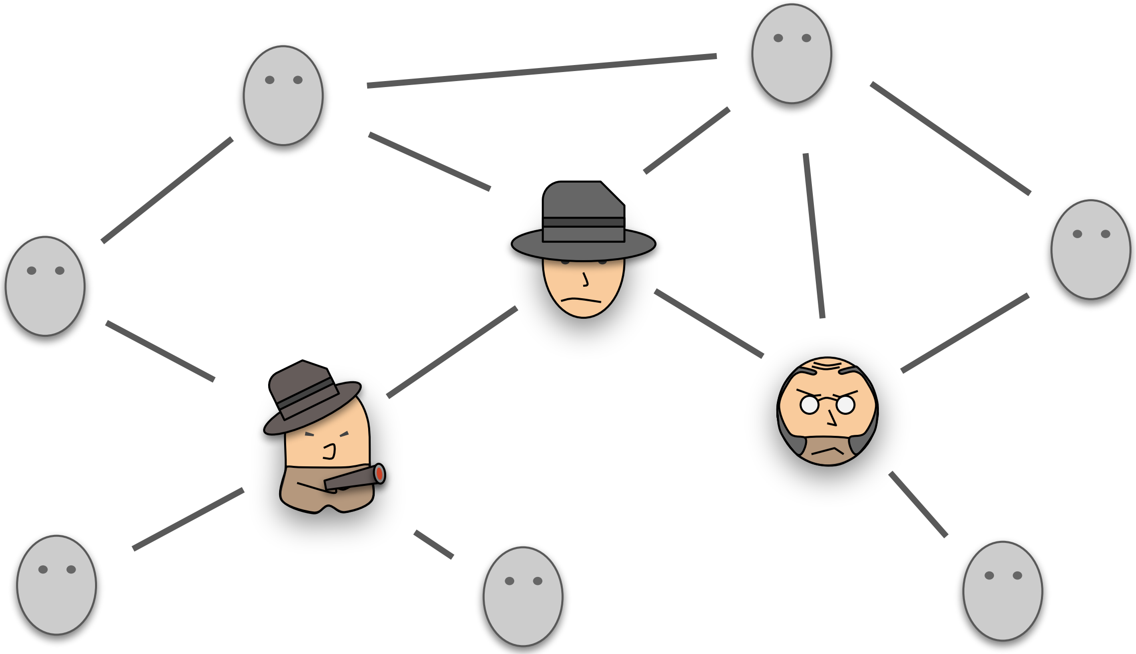
Closeness

1 over the average distance

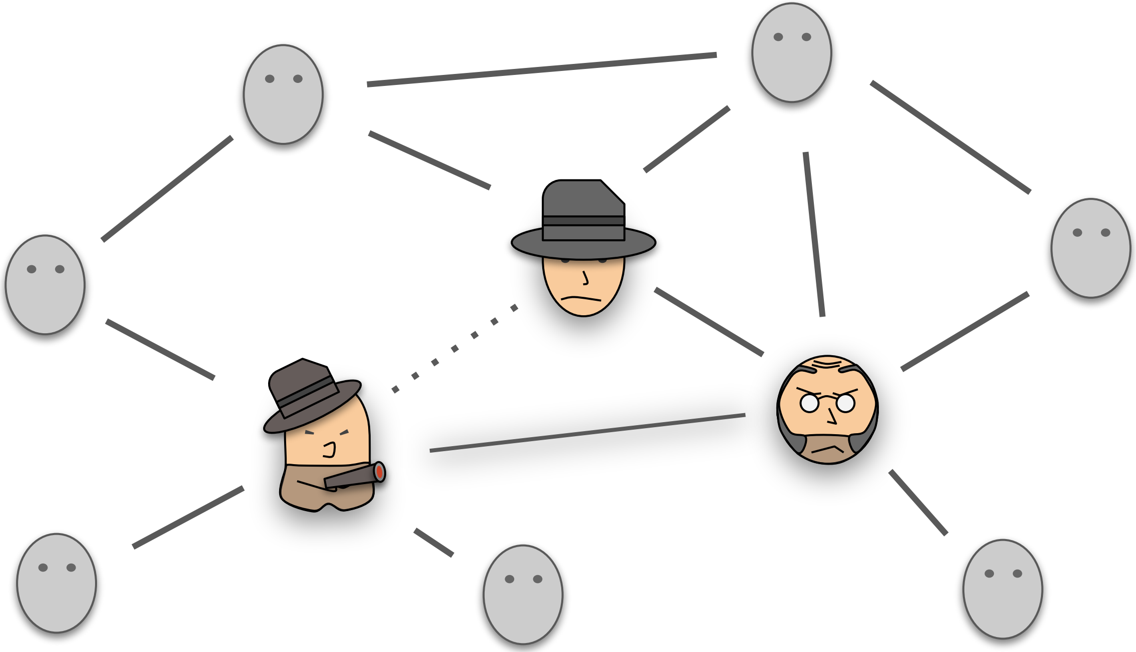
Investigation of criminal networks



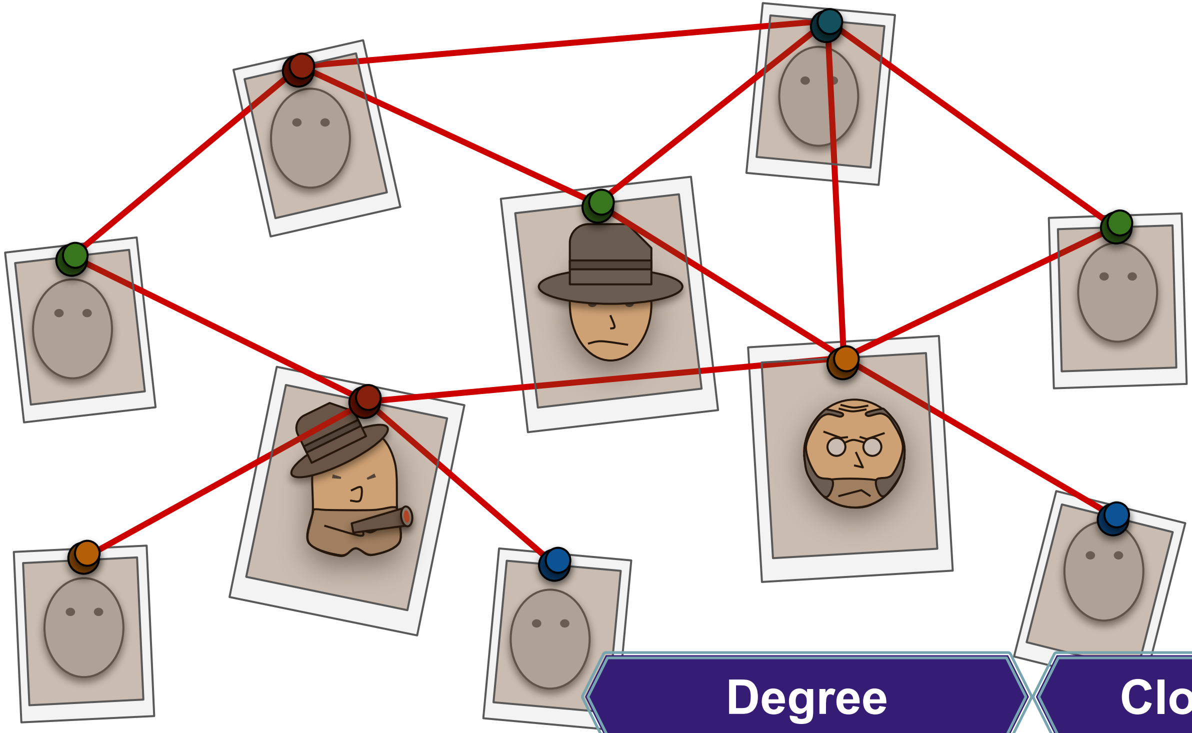
Investigation of criminal networks

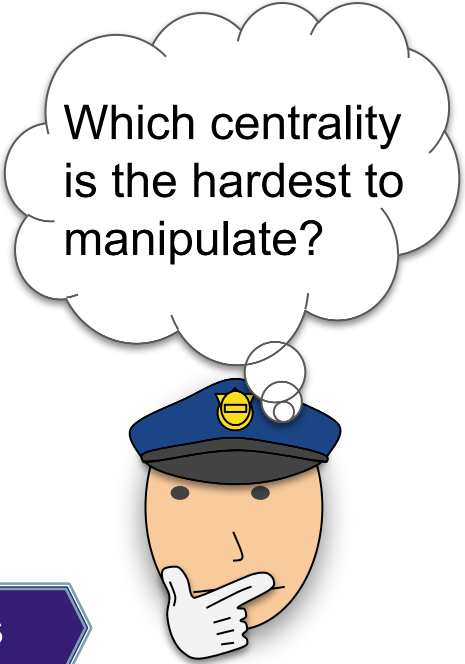


Investigation of criminal networks



Investigation of criminal networks





Degree

Closeness

Betweenness

Eigenvector

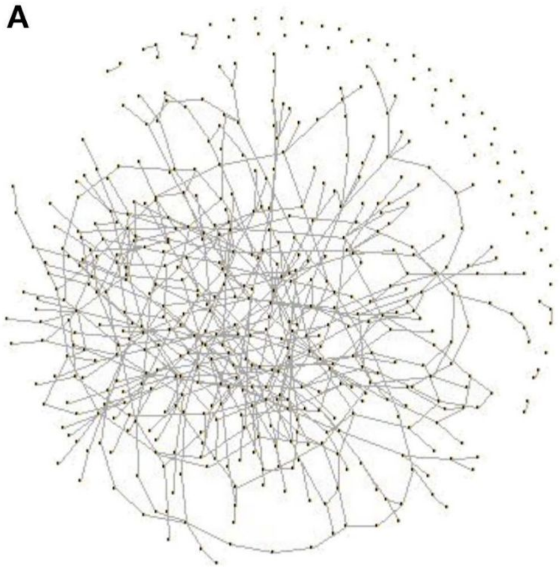
Setting

Setting: Measure of Manipulability

$$M(\mathcal{G}, v, F, \mathcal{A})$$

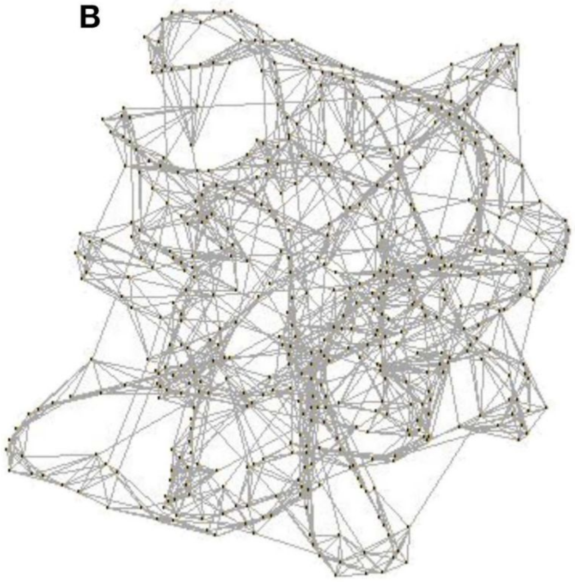
- $\mathcal{G}$ - graph distribution
- v - evader node
- F - centrality measure
- $\mathcal{A}$ - action function

Setting: Graph Distribution $\mathcal{G}$



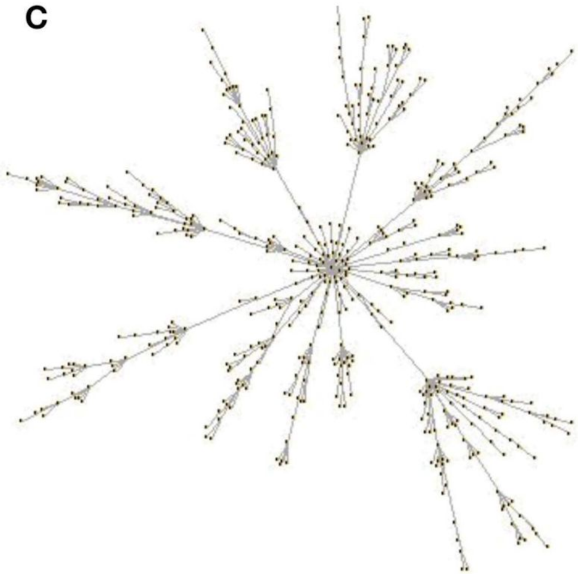
(A) Erdős-Rényi

Random Graphs



(B) Watts-Strogatz

Small World Network

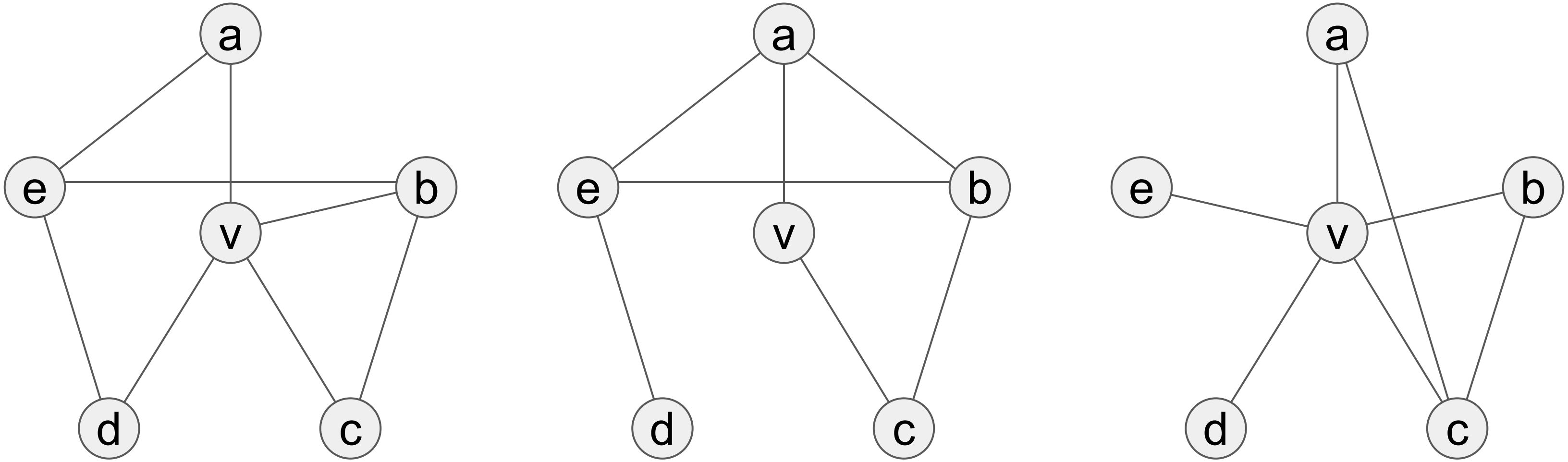


(C) Barabási–Albert

Preferential Attachment Network

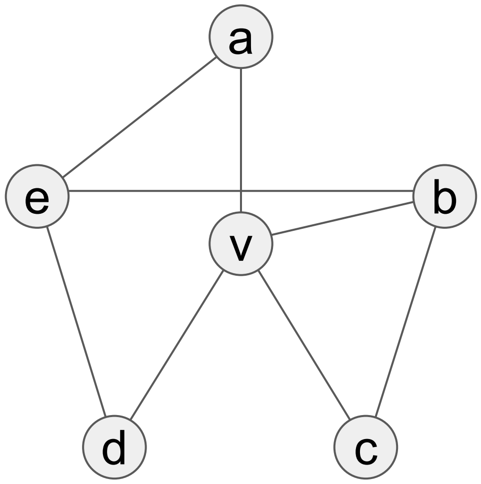
Setting: Graph Distribution

$$\mathcal{G}$$



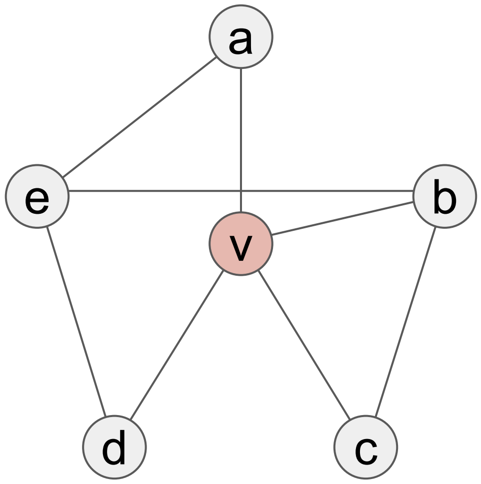
$\mathbb{P}_{\mathcal{G}}(G = G_0) = 3.7\%$ $\mathbb{P}_{\mathcal{G}}(G = G_1) = 6.4\%$ $\mathbb{P}_{\mathcal{G}}(G = G_2) = 0.8\%$

Setting: Evader & Actions



node	Degree	Closeness
v	I (4)	I (1 / 6)
a	IV (2)	IV (1 / 8)
b	II (3)	II (1 / 7)
c	IV (2)	VI (1 / 9)
d	IV (2)	IV (1 / 8)
e	II (3)	II (1 / 7)

Setting: Evader & Actions



node	Degree	Closeness
v	I (4)	I (1 / 6)
a	IV (2)	IV (1 / 8)
b	II (3)	II (1 / 7)
c	IV (2)	VI (1 / 9)
d	IV (2)	IV (1 / 8)
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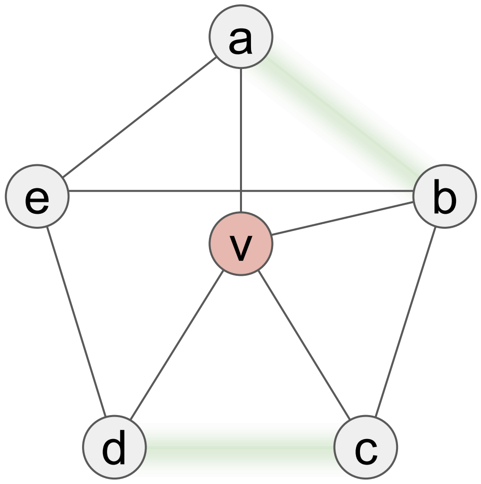
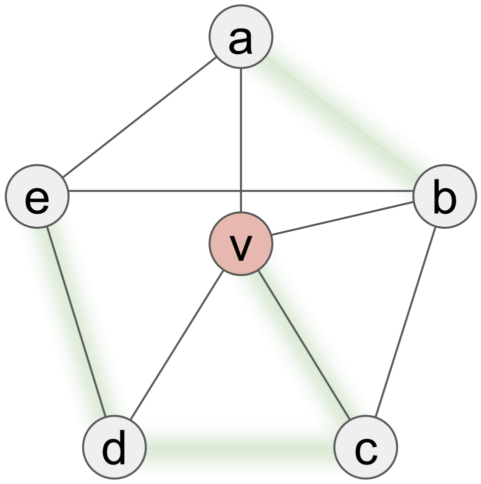
Setting: Evader & Actions



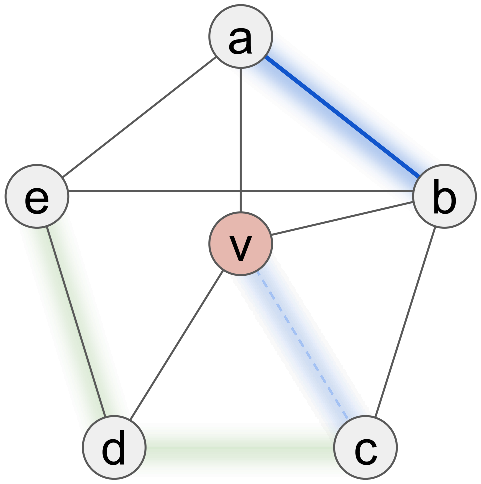
node	Degree	Closeness
v	I (4)	I (1 / 6)
a	IV (2)	IV (1 / 8)
b	II (3)	II (1 / 7)
c	IV (2)	VI (1 / 9)
d	IV (2)	IV (1 / 8)
e	II (3)	II (1 / 7)

Setting: Evader & Actions



node	Degree	Closeness
v	I (4)	I (1 / 6)
a	IV (2)	IV (1 / 8)
b	II (3)	II (1 / 7)
c	IV (2)	VI (1 / 9)
d	IV (2)	IV (1 / 8)
e	II (3)	II (1 / 7)

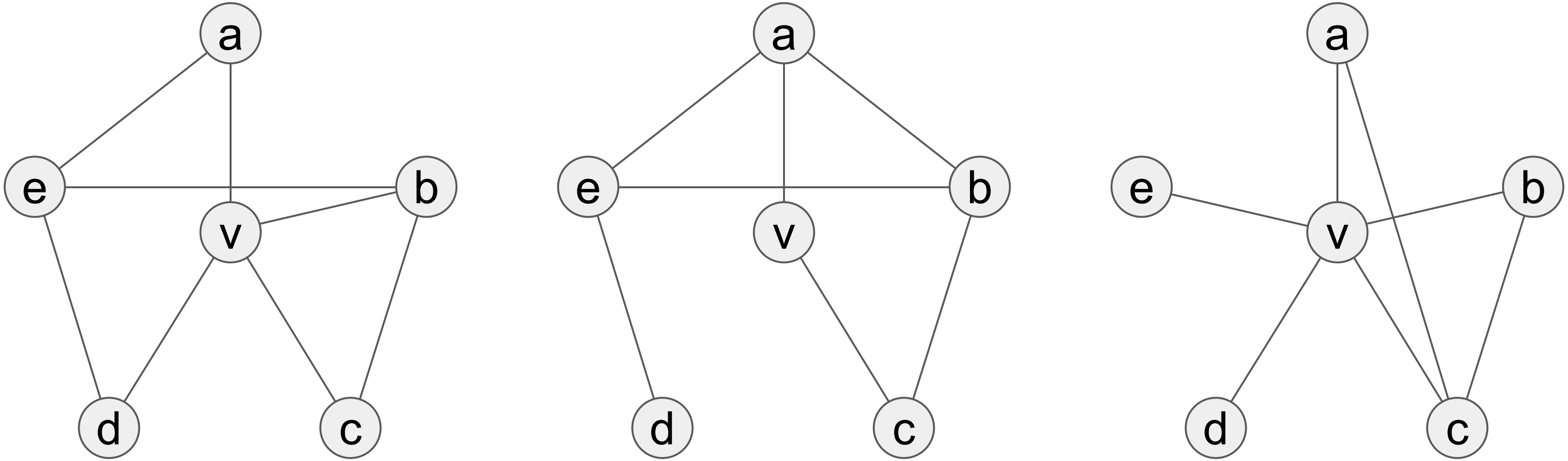
Setting: Evader & Actions



node	Degree	Closeness
v	II (3)	II (1 / 7)
a	II (3)	II (1 / 7)
b	I (4)	I (1 / 6)
c	VI (1)	VI (1 / 10)
d	V (2)	V (1 / 9)
e	II (3)	II (1 / 7)

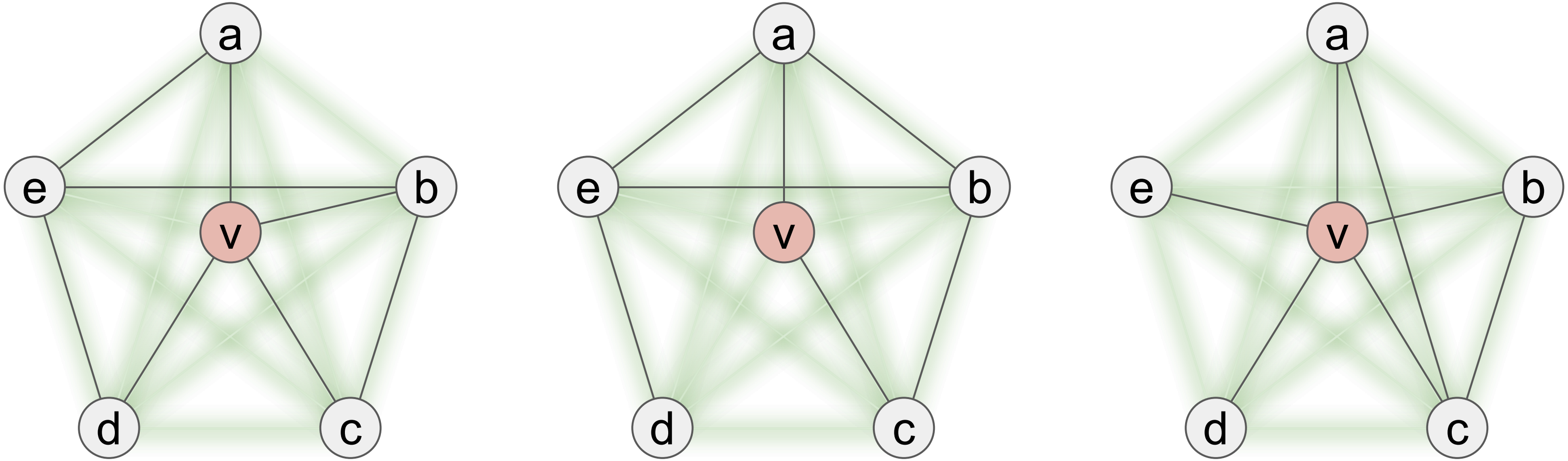
Setting: Action function

$$\mathcal{A}(G) = \{\text{ allowed actions in graph } G\}$$



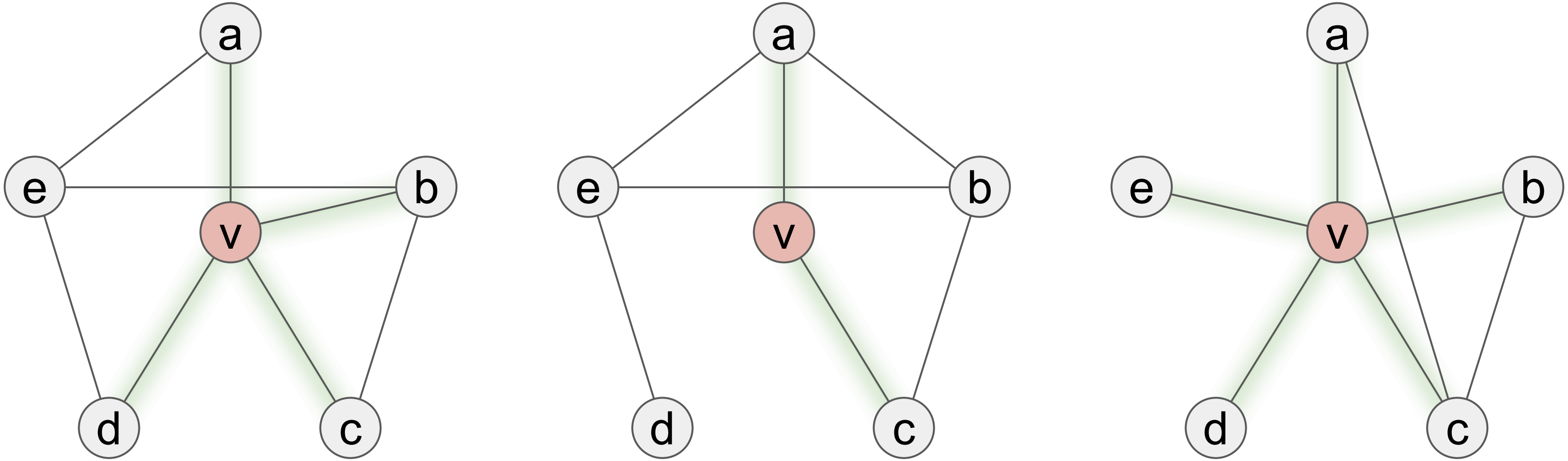
e.g.: All changes

$$\mathcal{A}_1(G) = \{a \subseteq V : |a| = 2\}$$



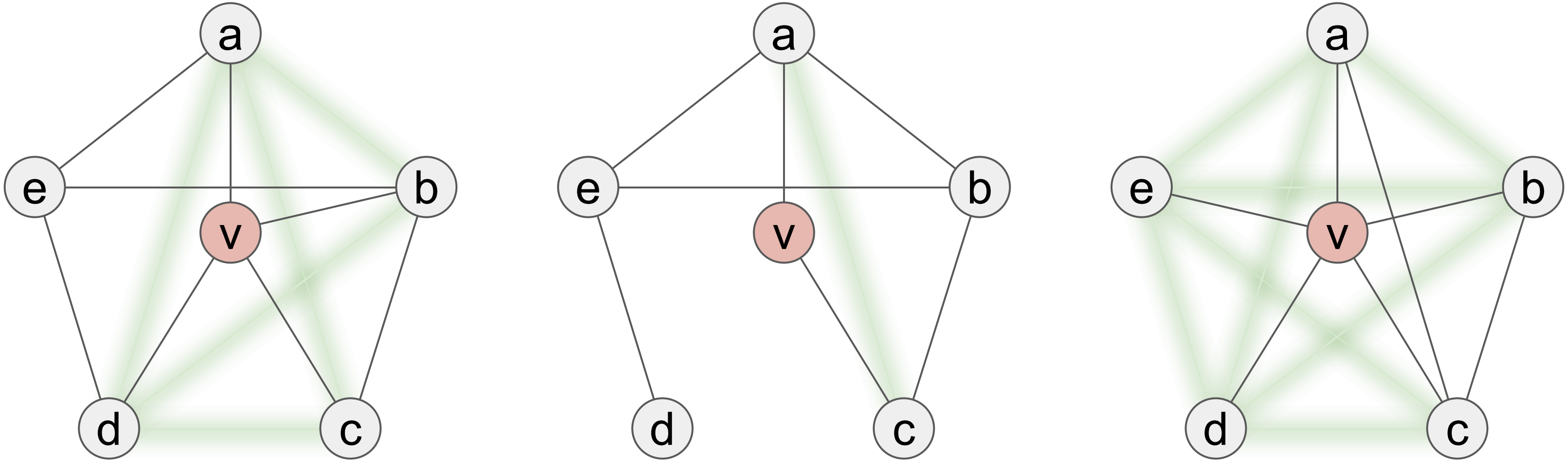
e.g.: Remove Neighbors

$$\mathcal{A}_2(G) = \{a \in E[G] : v \in a\}$$



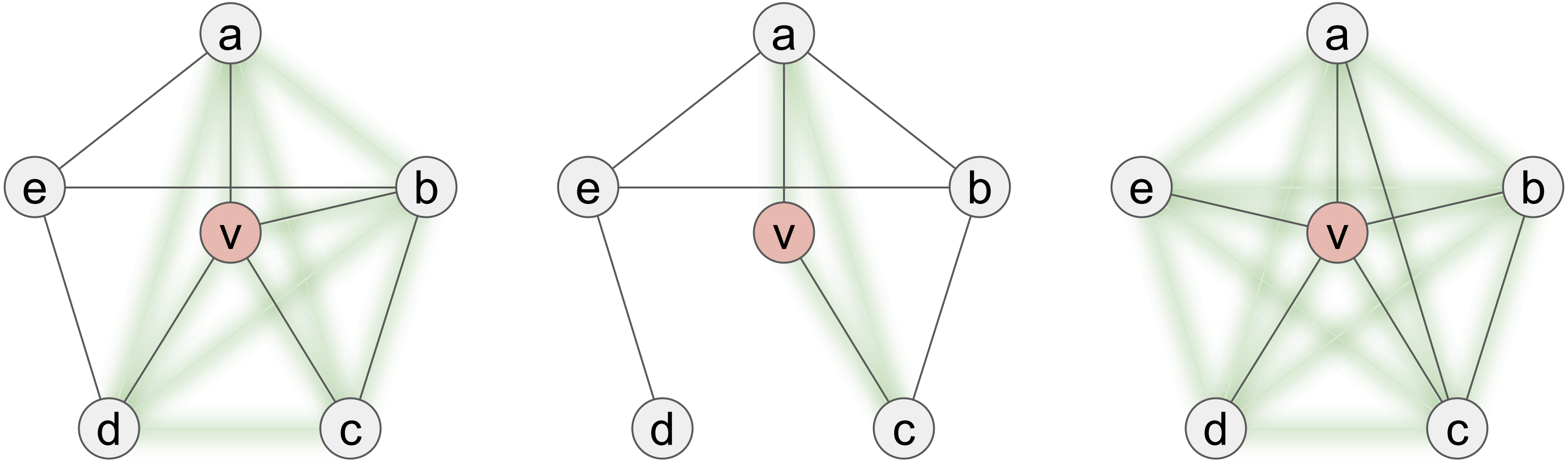
e.g.: Add Between Neighbors

$$\mathcal{A}_3(G) = \{a \subseteq \mathcal{N}_G(v) : v \in a \wedge a \notin E[G]\}$$



e.g.: Local changes

$$\mathcal{A}_4(G) = \{a \in E[G] : v \in a\} \cup \{a \subseteq \mathcal{N}_G(v) : v \in a\}$$



Setting: Measure of Manipulability

$$M(\mathcal{G}, v, F, \mathcal{A}) \in [0, 1]$$

- $\mathcal{G}$ - graph distribution
- v - evader node
- F - centrality measure
- $\mathcal{A}$ - action function

1 Very easy to manipulate

0 Very hard to manipulate

AMAR Measure of Manipulability

Axiomatic Approach

Axioms for Measure of Manipulability:

- Unmanipulability
- Full Manipulability
- Weak Dominance
- Redundant Action
- Neutrality
- Linearity
- Normalisation

Axiomatic Approach

Axioms for Measure of Manipulability:

- Unmanipulability
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If it is certain that it is impossible to hide the evader with any subset of allowed actions, then manipulability is equal to 0

Axiomatic Approach

Axioms for Measure of Manipulability:

- Unmanipulability
- Full Manipulability
- Weak Dominance
- Redundant Action
- Neutrality
- Linearity
- Normalisation

If it is certain that any subset of actions that hides the evader according to one centrality measure, hides it also according to the other, then the latter measure is at least as manipulable as the former

Axiomatic Approach

Axioms for Measure of Manipulability:

- Unmanipulability
- Full Manipulability
- Weak Dominance
- Redundant Action
- Neutrality
- Linearity
- Normalisation

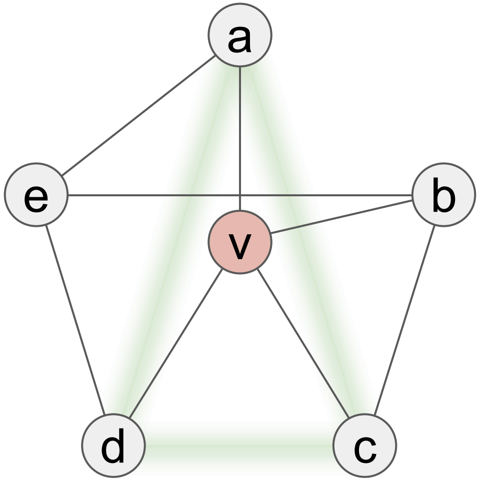
Main Theorem:
A measure of manipulability satisfies all seven axioms if and only if it is the AMAR Measure of Manipulability

MAR measure

MAR (Minimal Actions Required) =
1 over the smallest number of actions
that hides the evader
or 0 if it is impossible to hide

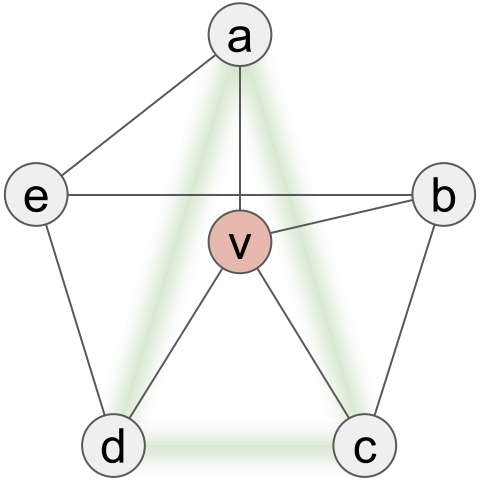
$$MAR(G, v, F, A) \in [0, 1]$$

MAR measure



node	Degree
v	I (4)
a	IV (2)
b	II (3)
c	IV (2)
d	IV (2)
e	II (3)

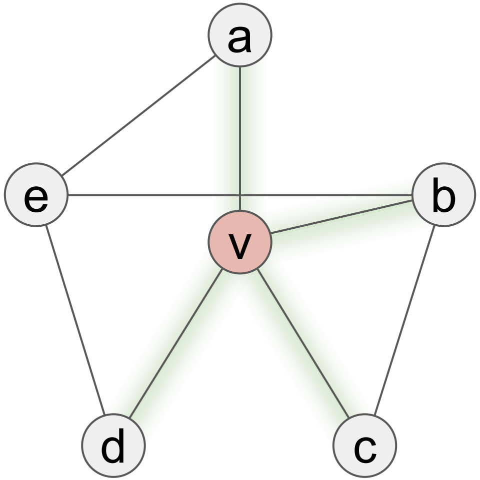
MAR measure



node	Degree
v	I (4)
a	IV (2)
b	II (3)
c	IV (2)
d	IV (2)
e	II (3)

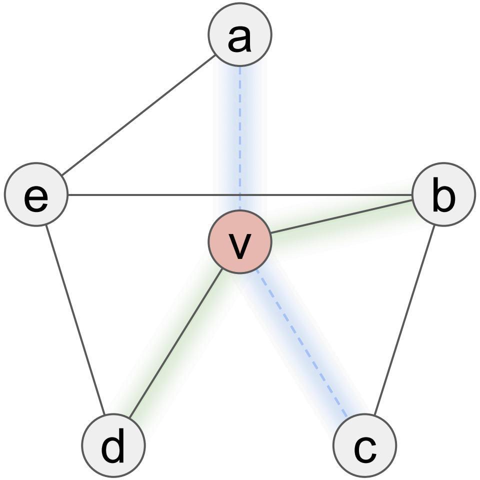
$$MAR(G, v, D, A) = 0$$

MAR measure



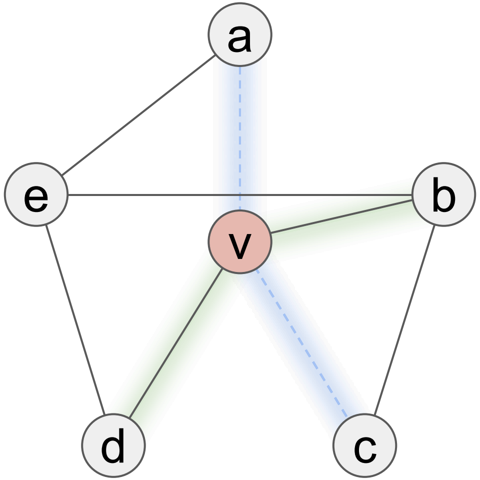
node	Degree
v	I (4)
a	IV (2)
b	II (3)
c	IV (2)
d	IV (2)
e	II (3)

Impact set



node	Degree
v	III (2)
a	V (1)
b	I (3)
c	V (1)
d	III (2)
e	I (3)

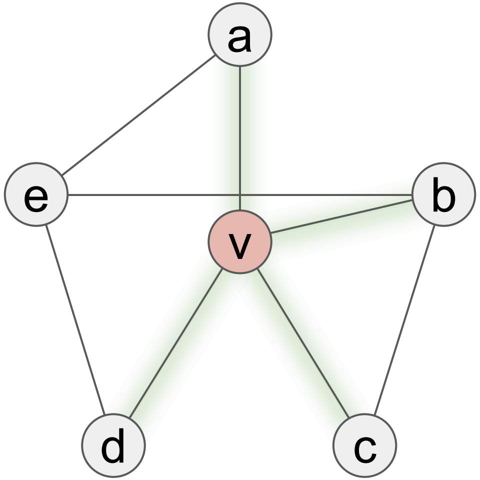
Impact set



node	Degree
v	III (2)
a	V (1)
b	I (3)
c	V (1)
d	III (2)
e	I (3)

$$MAR(G, v, D, A) = 1/2$$

MAR measure



node	Degree	Closeness
v	I (4)	I (1 / 6)
a	IV (2)	IV (1 / 8)
b	II (3)	II (1 / 7)
c	IV (2)	VI (1 / 9)
d	IV (2)	IV (1 / 8)
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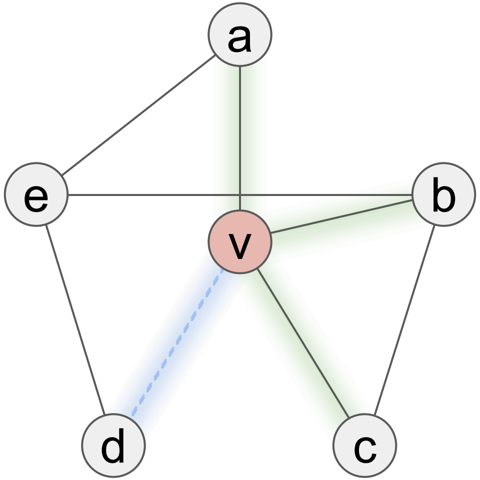
$$MAR(G, v, D, A) = 1/2$$

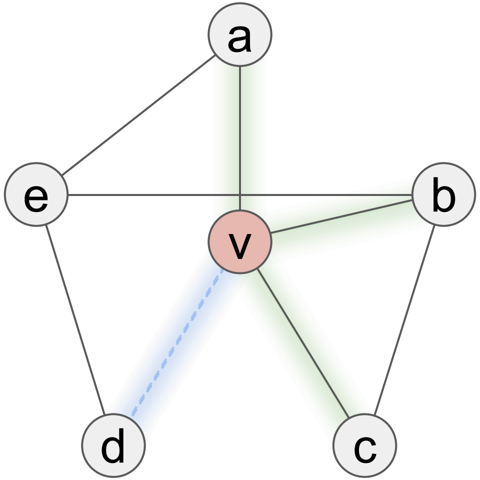
MAR measure



node	Degree	Closeness
v	I (4)	III (1 / 8)
a	IV (2)	IV (1 / 8)
b	II (3)	I (1 / 7)
c	IV (2)	V (1 / 10)
d	IV (2)	VI (1 / 11)
e	II (3)	I (1 / 7)

$$MAR(G, v, D, A) = 1/2$$

MAR measure



node	Degree	Closeness
v	I (4)	III (1 / 8)
a	IV (2)	IV (1 / 8)
b	II (3)	I (1 / 7)
c	IV (2)	V (1 / 10)
d	IV (2)	VI (1 / 11)
e	II (3)	I (1 / 7)

$$MAR(G, v, D, A) = 1/2 \qquad MAR(G, v, C, A) = 1$$

AMAR measure

Averaged Minimal Actions Required

$$AMAR(\mathcal{G}, v, F, \mathcal{A}) = \mathbb{E}_{\mathcal{G}}\big(MAR(G, v, F, \mathcal{A}(G))\big)$$

Evaluation

Evaluation of AMAR

4 Centralities:

- Degree
- Closeness
- Betweenness
- Eigenvector

Evaluation of AMAR

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- Degree
- Closeness
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4 Graph Distributions:

- Random Graphs
- Small-World
- Preferential Attachment
- Cellular Networks

Evaluation of AMAR

4 Centralities:

- Degree
- Closeness
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4 Graph Distributions:

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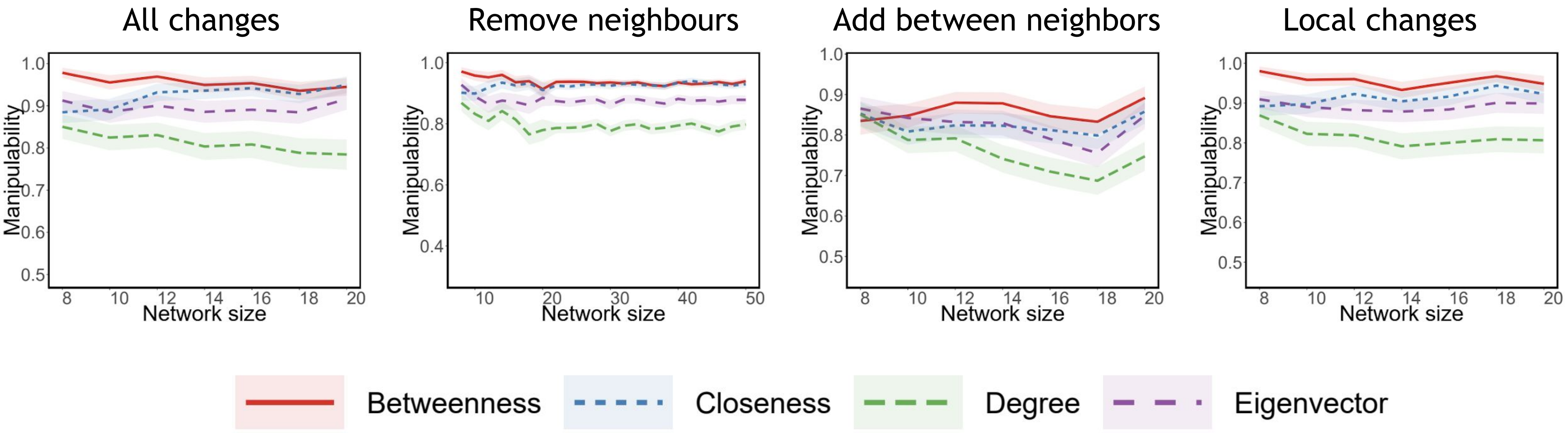
- Cellular Networks

4 Action functions:

- All changes
- Remove neighbours
- Add between neighbors
- Local changes

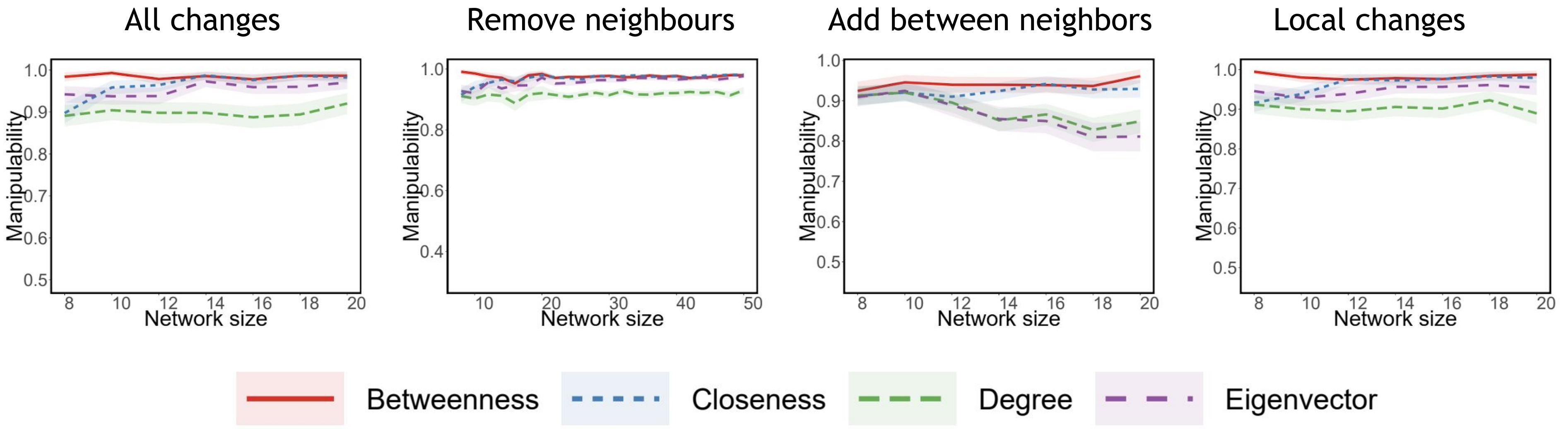
Evaluation of AMAR

Random Graphs - Erdős-Rényi model



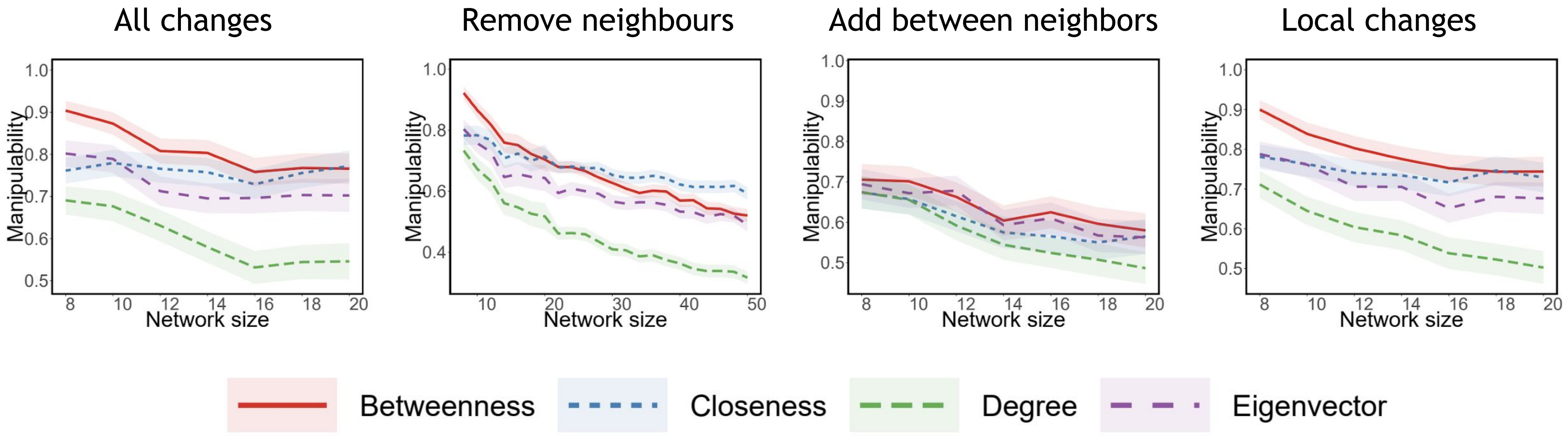
Evaluation of AMAR

Small-world networks - Watts-Strogatz model



Evaluation of AMAR

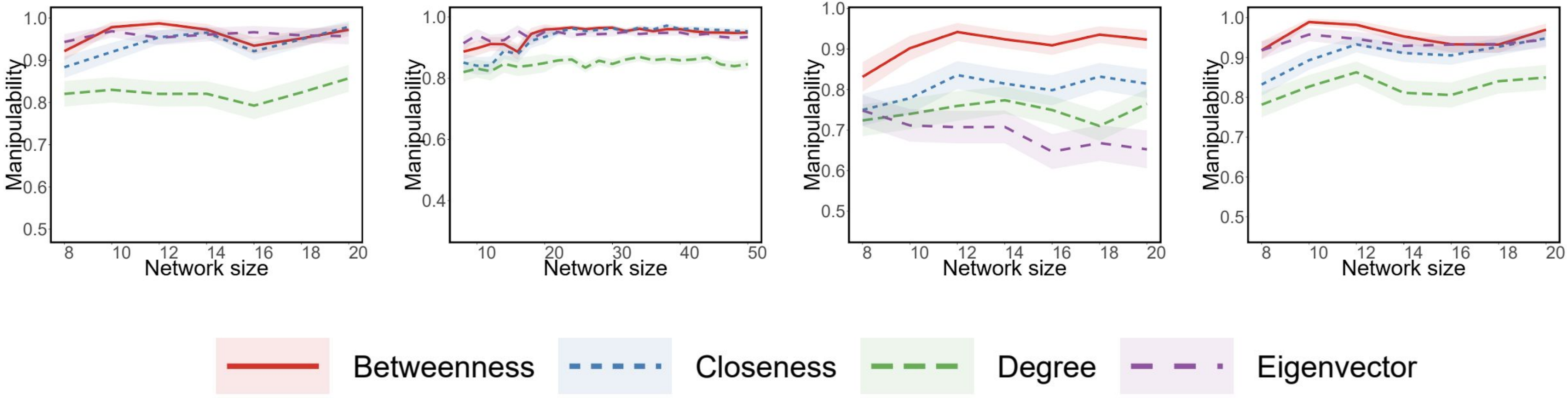
Preferential attachment networks - Barabási-Albert model



Evaluation of AMAR

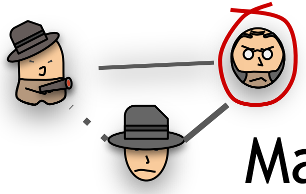
Cellular networks (Tsvetovat and Carley, 2005)

All changes | Remove neighbours | Add between neighbors | Local changes



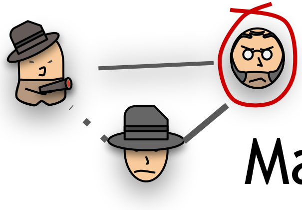
Summary

Summary



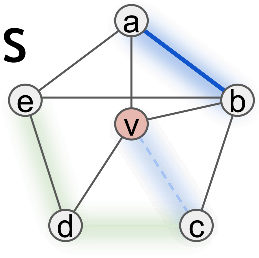
Manipulation of
Centrality measures

Summary

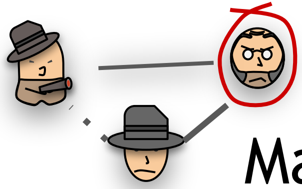


Manipulation of
Centrality measures

AMAR = Averaged
Minimal Actions
Required

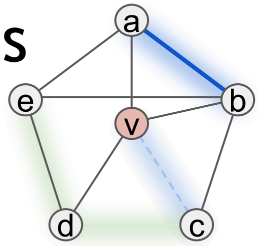


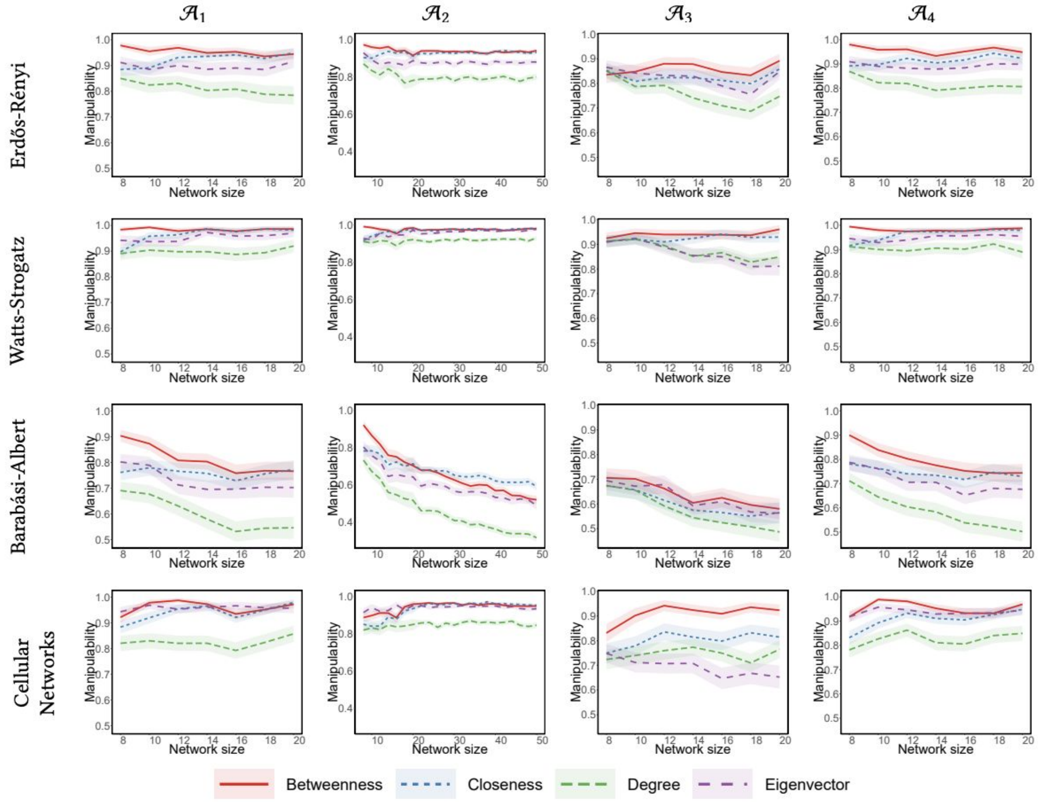
Summary



Manipulation of Centrality measures

AMAR = Averaged Minimal Actions Required





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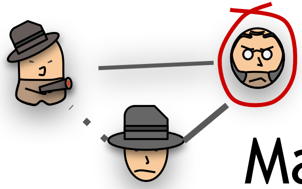

Manipulation of Centrality measures

AMAR = Averaged Minimal Actions Required