Agent Ontology Alignment Repair through Dynamic Epistemic Logic

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Agents communicate using alignments between ontologies.

Repair alignments when failures occur via adaptation operators.

We introduce a formal framework to model this and prove that some adaptation operators are incorrect or redundant, and all are incomplete.
Agents communicate using alignments between ontologies.
Summary

- Agents communicate using alignments between ontologies
- Repair alignments when failures occur via adaptation operators
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We introduce a formal framework to model this
Agents communicate using alignments between ontologies
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We introduce a formal framework to model this
And prove that some adaptation operators are incorrect or redundant, and all are incomplete
Alignment Repair Game [Euzenat 2014, Euzenat 2017]
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- Agents use (different) ontologies
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

- Agents use (different) ontologies

\[ O_a \]

\[
\begin{aligned}
&\equiv \\
&\oplus \\
&\subseteq \\
&\equiv \\
Black_a &\rightarrow &\equiv &\equiv &\equiv \\
\equiv &\rightarrow &\oplus &\subseteq \\
Small_a &\oplus &\rightarrow &\equiv &\equiv \\
\equiv &\rightarrow &\oplus &\subseteq &\equiv \\
Large_a &\rightarrow &\equiv &\equiv &\equiv \\
Small_a &\oplus &\rightarrow &\equiv &\equiv \\
\equiv &\rightarrow &\oplus &\subseteq &\equiv \\
Large_a &\rightarrow &\equiv &\equiv &\equiv
\end{aligned}
\]
Agents use (different) ontologies
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

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- Agents use (different) ontologies
- They use alignments to communicate

\[ \begin{align*}
&\text{Agents use (different) ontologies} \\
&\text{They use alignments to communicate}
\end{align*} \]
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

- Agents use (different) ontologies
- They use alignments to communicate
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

- Alignments are computed by ontology matching tools

```latex
\begin{tikzpicture}
  \node (Oa) at (0,0) {$O_a$};
  \node (Ob) at (7,0) {$O_b$};
  \node (Ta) at (-2,-2) {$T_a$};
  \node (Tb) at (5,-2) {$T_b$};
  \node (Sa) at (-4,-4) {$Small_a$};
  \node (Ls) at (-2,-4) {$Large_a$};
  \node (Wa) at (0,-4) {$White_a$};
  \node (Sb) at (3,-4) {$Small_b$};
  \node (Lb) at (5,-4) {$Large_b$};
  \node (Sz) at (2,-4) {$Square_b$};
  \node (Tz) at (3,-6) {$Triangle_b$};
  \node (Sz1) at (1.5,-5.5) {$\triangledown$, $\triangle$};

  \draw[->, dashed] (Oa) to (Ta);
  \draw[->, dashed] (Oa) to (Wa);
  \draw[->, dashed] (Oa) to (Sa);
  \draw[->, dashed] (Oa) to (Ls);
  \draw[->, dashed] (Ob) to (Tb);
  \draw[->, dashed] (Ob) to (Lb);
  \draw[->, dashed] (Ob) to (Sz);
  \draw[->, dashed] (Ob) to (Sz1);
  \draw[->, dashed] (Ta) to (Sa);
  \draw[->, dashed] (Ta) to (Sz);
  \draw[->, dashed] (Ta) to (Tz);
  \draw[->, dashed] (Ta) to (Sb);
  \draw[->, dashed] (Ta) to (Ls);
  \draw[->, dashed] (Wa) to (Sb);
  \draw[->, dashed] (Wa) to (Lb);
  \draw[->, dashed] (Wa) to (Sz);
  \draw[->, dashed] (Wa) to (Sz1);
  \draw[->, dashed] (Sa) to (Ls);
  \draw[->, dashed] (Sa) to (Sz);

  \node (a) at (-6,0) {$O_a$};
  \node (b) at (6,0) {$O_b$};
  \node (c) at (-2,-2) {$T_a$};
  \node (d) at (5,-2) {$T_b$};
  \node (e) at (-4,-4) {$Small_a$};
  \node (f) at (-2,-4) {$Large_a$};
  \node (g) at (0,-4) {$White_a$};
  \node (h) at (3,-4) {$Small_b$};
  \node (i) at (5,-4) {$Large_b$};
  \node (j) at (2,-4) {$Square_b$};
  \node (k) at (3,-6) {$Triangle_b$};
  \node (l) at (1.5,-5.5) {$\triangledown$, $\triangle$};

  \draw[->, dashed] (a) to (e);
  \draw[->, dashed] (a) to (f);
  \draw[->, dashed] (a) to (g);
  \draw[->, dashed] (b) to (h);
  \draw[->, dashed] (b) to (i);
  \draw[->, dashed] (b) to (j);
  \draw[->, dashed] (c) to (e);
  \draw[->, dashed] (c) to (f);
  \draw[->, dashed] (c) to (g);
  \draw[->, dashed] (d) to (h);
  \draw[->, dashed] (d) to (i);
  \draw[->, dashed] (d) to (j);
  \draw[->, dashed] (c) to (e);
  \draw[->, dashed] (c) to (f);
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\end{tikzpicture}
```
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

- Alignments are computed by ontology matching tools
- Alignments may be incorrect or incomplete
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

- Alignments are computed by ontology matching tools
- Alignments may be incorrect or incomplete
- Alignment Repair Game (ARG)

![Alignment Repair Game Diagram]

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Alignment Repair through DEL
Alignment Repair Game [Euzenat 2014, Euzenat 2017]
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

Diagram:

- $D_a$ connected to $C_a$
- $C_b$ connected to $D_b$
- $+$ symbols denote alignment repair operations:
  - $X$ (add)
  - $\triangleright$ (addjoin)
  - $\triangleright$ (refine)
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

\[ D_a \quad \sqcup \quad C_a \quad \equiv \quad C_b \quad \equiv \quad D_b \]
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

Diagram:

```
D_a  C_a  C_b  D_b

O
```

Discussion:

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Alignment Repair through DEL
Alignment Repair Game [Euzenat 2014, Euzenat 2017]
Alignment Repair Game [Euzenat 2014, Euzenat 2017]

Diagram showing the relationship between $D_a$, $C_a$, $C_b$, and $D_b$ with annotations for addition and addition join.
Alignment Repair Game [Euzenat 2014, Euzenat 2017]
Alignment Repair Game [Euzenat 2014, Euzenat 2017]
Objective

Experimental results show that if agents play ARG: agents converge towards successful communication, and improve their alignments. But it is an open question whether the adaptation operators are formally correct, complete or redundant?
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  - and improve their alignments.
- But it is an open question whether the adaptation operators are **formally correct, complete or redundant**?
We introduce a formal framework based on DEL that
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- models the adaptation operators.
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- expresses ontologies and alignments,
- models the adaptation operators.

To formally establish the correctness, partial redundancy and incompleteness of the adaptation operators.
Definition (DEOL Syntax)

The syntax, $\mathcal{L}_{DEOL}$, of (multi-agent) DEOL is defined by:

$$\phi ::=$$
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$$\phi ::= C(o) \mid CRD$$

$$R \in \{\sqsubseteq, \equiv, \oplus\}$$
Definition (DEOL Syntax)

The syntax, $\mathcal{L}_{DEOL}$, of (multi-agent) DEOL is defined by:

$$\phi ::= C(o) \mid CRD \mid \phi \land \psi \mid \neg \phi$$

$$R \in \{\sqsubseteq, \equiv, \oplus\}$$
Definition (DEOL Syntax)

The syntax, $\mathcal{L}_{DEOL}$, of (multi-agent) DEOL is defined by:

$$\phi ::= C(o) \mid CRD \mid \phi \land \psi \mid \neg \phi \mid K_a \phi \mid B_a \phi$$

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Dynamic Epistemic Ontology Logic (DEOL): Syntax

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The syntax, $\mathcal{L}_{DEOL}$, of (multi-agent) DEOL is defined by:

$$\phi ::= C(o) \mid CRD \mid \phi \land \psi \mid \neg \phi \mid K_a \phi \mid B_a \phi \mid [\dagger \phi] \psi$$

$$R \in \{\sqsubseteq, \equiv, \oplus\}, \quad \dagger \in \{!, \uparrow\}$$
Dynamic Epistemic Ontology Logic (DEOL): Models
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\[ \text{White}(\text{△}) \]
\[ \text{Black}(\text{▲}) \]

\[ \text{White}(\text{△}) \]
\[ \text{White}(\text{▲}) \]
Dynamic Epistemic Ontology Logic (DEOL): Models

\[ \text{White}(\triangleright) \]
\[ \text{Black}(\blacktriangle) \]

\[ \text{White}(\triangleright) \]
\[ \text{White}(\blacktriangle) \]
Dynamic Epistemic Ontology Logic (DEOL): Models

\[ K_a \text{White}(\triangle) \quad \text{“Agent } a \text{ knows that } \triangle \text{ belongs to the class White”} \]
Dynamic Epistemic Ontology Logic Logic (DEOL): Models

\[ B_a \text{White}(\blacktriangle) \] “Agent a believes that \blacktriangle belongs to the class White”
[!\text{Black}(\blacktriangle)] K_a \text{Black}(\blacktriangle) \text{ “After announcing !Black(\blacktriangle), a knows this”}
Dynamic Epistemic Ontology Logic (DEOL): Models

\[ !\text{Black}(\blacktriangle) K_a \text{Black}(\blacktriangle) \] "After announcing !\text{Black}(\blacktriangle), a knows this"
[!Black(▲)]K_a Black(▲) “After announcing !Black(▲), a knows this”
Dynamic Epistemic Ontology Logic (DEOL): Models

\[ a \Rightarrow \begin{array}{c} \text{White}(\triangle) \\
\text{Black}(\blacktriangle) \end{array} \xrightarrow{a} \begin{array}{c} \text{White}(\triangle) \\
\text{White}(\blacktriangle) \end{array} \]

\[ [\uparrow \text{Black}(\blacktriangle)] B_a \text{Black}(\blacktriangle) \] “After upgrading with \([\uparrow \text{Black}(\blacktriangle)]\), \(a\) believes this”
Dynamic Epistemic Ontology Logic (DEOL): Models

\[ [\uparrow \text{Black}(\blacktriangle)] B_a \text{Black}(\blacktriangle) \] “After upgrading with \( \uparrow \text{Black}(\blacktriangle) \), \( a \) believes this”
Dynamic Epistemic Ontology Logic Logic (DEOL): Models

\[ [\uparrow \text{Black}(\Box)] B_a \text{Black}(\Box) \text{ “After upgrading with } \uparrow \text{Black}(\Box), a \text{ believes this”} \]
Dynamic Epistemic Ontology Logic (DEOL): Semantics
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\[ M, w \models C(x) \iff x \in C^I_w \]

\[ M, w \models C \sqsubseteq D \iff C^I_w \subseteq D^I_w \]

\[ M, w \models C \equiv D \iff C^I_w = D^I_w \]

\[ M, w \models C \oplus D \iff C^I_w \cap D^I_w = \emptyset \]
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\[ \mathcal{M}, w \models C \oplus D \iff C^I_w \cap D^I_w = \emptyset \]
\[ \mathcal{M}, w \models \phi \land \psi \iff \mathcal{M}, w \models \phi \text{ and } \mathcal{M}, w \models \psi \]
\[ \mathcal{M}, w \models \neg \phi \iff \mathcal{M}, w \not\models \phi \]
Dynamic Epistemic Ontology Logic (DEOL): Semantics

\[ M, w \models C(x) \iff x \in C^I_w \]
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\[ M, w \models \neg \phi \iff M, w \not\models \phi \]
\[ M, w \models K_a \phi \iff \forall v \text{ s.t. } w \sim_a v : M, v \models \phi \]
\[ M, w \models B_a \phi \iff \forall v \text{ s.t. } w \rightarrow_a v : M, v \models \phi \]
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\( M, w \models B_a \phi \iff \forall v \text{ s.t. } w \rightarrow_a v : M, v \models \phi \)

\( M, w \models [! \phi] \psi \iff M !^\phi, w \models \psi \)

\( M, w \models [\uparrow \phi] \psi \iff M \uparrow^\phi, w \models \psi \)
Translation

ARG state (s)
ARG state (s) \xrightarrow{z} \text{DEOL axioms}
ARG state (s) $\xrightarrow{z}$ DEOL axioms $\xrightarrow{I}$ DEOL models ($\mathcal{M}$)
 ARG state \((s)\) \xrightarrow{z} \text{DEOL axioms} \xrightarrow{I} \text{DEOL models} (\mathcal{M})

\[\downarrow \alpha\]

ARG state \((\alpha(s))\)
ARG state \((s)\) \(\xrightarrow{z} \) DEOL axioms \(\xrightarrow{I} \) DEOL models \((\mathcal{M})\)

\[
\begin{align*}
\text{ARG state } (\alpha(s)) & \xrightarrow{\delta} \delta(\alpha) \\
\downarrow \alpha & \xrightarrow{\delta} \delta(\alpha) \\
\text{DEOL axioms} & \xrightarrow{I} \text{DEOL models } (\mathcal{M}^{\delta(\alpha)})
\end{align*}
\]
ARG state \((s)\) \(\xrightarrow{Z}\) DEOL axioms \(\xrightarrow{I}\) DEOL models \((\mathcal{M})\)

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Semantics of ARG states

ARG state \((s)\) \(\xrightarrow{z}\) DEOL axioms \(\xrightarrow{I}\) DEOL models \((M)\)

\[
\begin{array}{c}
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\end{array}
\]
Semantics of ARG states

ARG state \((s)\) \(\xrightarrow{z} \) DEOL axioms \(\xrightarrow{I} \) DEOL models \((\mathcal{M})\)

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\begin{align*}
\alpha & \xrightarrow{\delta} \delta(\alpha) \\
\end{align*}
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ARG state \((\alpha(s))\) \(\xrightarrow{z} \) DEOL axioms \(\xrightarrow{I} \) DEOL models \((\mathcal{M}^{\delta(\alpha)})\)

Ontology Knowledge (OK) \(\mathcal{O}_a\) is known to agent \(a\)
Semantics of ARG states

 Ontology Knowledge (OK) $O_a$ is known to agent $a$
 Alignment Belief (AB) $A_{ab}$ is believed by agents $a$ and $b$
Semantics of ARG states

ARG state \((s)\) \(\xrightarrow{z} \) DEOL axioms \(\xrightarrow{I} \) DEOL models \((\mathcal{M})\)
\[
\begin{align*}
\alpha & \xrightarrow{\delta} \delta(\alpha) \\
\text{ARG state } & (\alpha(s)) \xrightarrow{z} \text{ DEOL axioms } \xrightarrow{I} \text{ DEOL models } (\mathcal{M}\delta(\alpha))
\end{align*}
\]

Ontology Knowledge (OK) \(\mathcal{O}_a\) is known to agent \(a\)
Alignment Belief (AB) \(A_{ab}\) is believed by agents \(a\) and \(b\)
Public Signature Awareness (PSA) All signatures are known to all agents
Dynamics

ARG state \((s)\) $\xrightarrow{\alpha} \delta \xrightarrow{\alpha} \delta(\alpha)$

DEOL axioms $\xrightarrow{\delta} \delta(\alpha)$

DEOL models \((M)\)

We model each round of ARG by:

$C \beta p \circ q$; if $\alpha C a p \circ q$ then $\delta p \alpha x C a, C b, \delta y p, p q$
Dynamics

\[
\begin{align*}
\text{ARG state } (s) & \xrightarrow{z} \text{DEOL axioms} \xrightarrow{I} \text{DEOL models } (\mathcal{M}) \\
\downarrow \alpha & \quad \quad \delta \quad \delta(\alpha) \\
\text{ARG state } (\alpha(s)) & \xrightarrow{z} \text{DEOL axioms} \xrightarrow{I} \text{DEOL models } (\mathcal{M}^{\delta(\alpha)})
\end{align*}
\]

We model each round of ARG by:
Dynamics

We model each round of ARG by:

\[ \text{!} \mathcal{C}_b(o); \text{ if } \neg \mathcal{C}_a(o) \text{ then } \delta(\alpha^{\mathcal{C}_a, \mathcal{C}_b, \equiv}(o)) \]
Operator Semantics

For failing correspondence $\langle C_a, C_b, \equiv \rangle \in A_{ab}$ with object $o$, the adaptation operators are defined on DEOL as follows:
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For failing correspondence $\langle C_a, C_b, \equiv \rangle \in A_{ab}$ with object $o$, the adaptation operators are defined on DEOL as follows:

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\delta(\text{delete}) = \uparrow (C_a \not\equiv C_b)
$$
Operator Semantics

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\delta(\text{add}) = \uparrow (C_a \not\equiv C_b \land C_a^{sup} \equiv C_b)
$$
For failing correspondence $\langle C_a, C_b, \sqsupseteq \rangle \in A_{ab}$ with object $o$, the adaptation operators are defined on DEOL as follows:

\[
\begin{align*}
\delta(\text{delete}) &= \uparrow (C_a \nsubseteq C_b) \\
\delta(\text{add}) &= \uparrow (C_a \nsubseteq C_b \land C_a^{sup} \sqsupseteq C_b) \\
\delta(\text{addjoin}) &= \uparrow (C_a \nsubseteq C_b \land C_a^{supO} \sqsupseteq C_b)
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Operator Semantics

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\delta(\text{addjoin}) &= \uparrow (C_a \not\equiv C_b \land C_a^{supO} \equiv C_b) \\
\delta(\text{refine}) &= \uparrow (C_a \not\equiv C_b \land \bigwedge \{C_a \equiv C_b^{sub}\})
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$$
\delta(\text{refine}) = \uparrow (C_a \not\equiv C_b \land \bigwedge \{ C_a \equiv \sub b \})
$$

$$
\delta(\text{refadd}) = \text{addjoin} \land \text{refine}
$$
Formal Properties of the Adaptation Operators
All but the add operator are correct
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- All but the add operator are correct
- delete, addjoin and refine are redundant for one agent
Formal Properties of the Adaptation Operators

- All but the `add` operator are correct
- `delete`, `addjoin` and `refine` are redundant for one agent
- All operators are incomplete
Formal Properties of the Adaptation Operators

- All **but the add operator** are correct
- delete, addjoin and refine are redundant for one agent
- **All operators are incomplete**
add is incorrect
Definition (Correctness)

Adaptation operator $\alpha$ is correct iff $\forall s: (z(s))^\delta(\alpha) \models z(\alpha(s))$. 

add is incorrect
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\[
\begin{align*}
\text{ARG state } (s) & \xrightarrow{z} \text{DEOL axioms} \\
\downarrow \alpha & \quad \quad \quad \quad \downarrow \delta(\alpha) \\
\text{ARG state } (\alpha(s)) & \xrightarrow{z} \text{DEOL axioms}
\end{align*}
\]
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\downarrow \alpha \quad \downarrow \delta(\alpha) \\
ARG state (\alpha(s)) \xrightarrow{z} \text{DEOL axioms}$
**add is incorrect**

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Adaptation operator $\alpha$ is correct iff $\forall s: (z(s))^{\delta(\alpha)} \models z(\alpha(s))$.

Proposition

The adaptation operator add is incorrect.
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Proof (sketch).
**add is incorrect**

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*The adaptation operator add is incorrect.*

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Adaptation operator $\alpha$ is correct iff $\forall s: (z(s))^{\delta(\alpha)} \models z(\alpha(s))$.

Proposition

The adaptation operator add is incorrect.

Proof (sketch).

\[
\begin{array}{c}
\text{(addjoin)} \\
\iff
\end{array}
\]

\[\begin{array}{c}
D_a \\
C_a \\
\text{X} \\
C_b \\
D_b
\end{array}
\]
All operators are incomplete
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Definition (Completeness)

Adaptation operator $\alpha$ is complete if and only if $\forall s: z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$. 
All operators are incomplete

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\[
\begin{align*}
\text{ARG state } (s) \xrightarrow{z} & \text{ DEOL axioms} \\
\downarrow^{\alpha} & \downarrow^{\delta(\alpha)} \\
\text{ARG state } (\alpha(s)) \xrightarrow{z} & \text{ DEOL axioms}
\end{align*}
\]
Formal Properties of the Adaptation Operators

All operators are incomplete

Definition (Completeness)

Adaptation operator $\alpha$ is complete if and only if $\forall s$: $z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$.
All operators are incomplete

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Adaptation operator $\alpha$ is complete if and only if $\forall s: z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$. 

\[
\begin{array}{ccc}
\text{ARG state } (s) & \xrightarrow{\alpha} & \text{DEOL axioms} \\
\downarrow \alpha & & \downarrow \delta(\alpha) \\
\text{ARG state } (\alpha(s)) & \xrightarrow{z} & \text{DEOL axioms}
\end{array}
\]
All operators are incomplete

Definition (Completeness)

Adaptation operator $\alpha$ is complete if and only if $\forall s: z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$.

Proposition

All adaptation operators are incomplete.
All operators are incomplete

Definition (Completeness)

Adaptation operator $\alpha$ is complete if and only if $\forall s: z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$.

Proposition

All adaptation operators are incomplete.

Proof.

After announcing $!C_b(o)$ the agents acquire knowledge $K_a(C_b(o))$ that is discarded in ARG.
Conclusion

We developed a theoretical framework for knowledge and belief evolution in ARG. Formally defined correctness, redundancy, and completeness. Some adaptation operators are incorrect or redundant, and all are incomplete. Agents in ARG do not possess full logical behavior.
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Thank you!