The Manipulability of Centrality Measures —An Axiomatic Approach

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1 INTRODUCTION

Centrality measures—methods for evaluating the nodes' relative positions and roles in the network—are among the most fundamental tools in social network analysis [21]. One of the issues that attracted attention in the literature is the *sensitivity* of centrality measures [8, 12]. This interest is driven by the fact that real-life data about the links in a network are often incomplete, erroneous, or otherwise distorted [16]. There are various reasons behind this, many of which are unintentional, such as the under-reporting of network relationships [24] (e.g., there are many real-life relationships that are not declared on Facebook) or the errors made by informants while asked about their ties [14]. The studies that evaluate the effects of such random distortions typically assume that only a certain percentage of the (randomly selected) links are known [6, 15, 17, 18] or there is a noise affecting the weights of the edges [22], and the analysis focuses on how the centrality-based ranking in such an incomplete or noisy network differs from the true one.

However, the sensitivity analysis based on random distortions is inadequate in situations where changes to the network do not occur by chance but rather as a a result of informed, rational decisions, i.e., due to *manipulation.* Since a straightforward *modus operandi* is to create fake accounts and/or add fake connections to boost the importance of certain network members and/or diminish others, various forms and magnitudes of manipulation are common in social networks [7, 11]. As a result, the interest in understanding how centrality measures can be manipulated has been recently growing in the literature. In particular, Crescenzi *et al.* [9] studied the problem of maximising Closeness centrality of a node by creating a limited amount of new edges incident to it. Analogous problems were also considered for Betweenness centrality [4], eccentricity [10, 20], and page-rank centralities [1, 19]. Also, Waniek *et al.* [25, 26] studied how an "evader" node could rewire a given number of edges in order to decrease her centrality.

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Since in most cases considered in the above literature obtaining an optimal solution turned out to be intractable, a typical approach was to develop a heuristic as opposed to an exact algorithm. The manipulability of a given centrality was then studied by comparing the ranking of nodes before and after applying the heuristic. Hence, manipulability was assessed in the context of a particular heuristic and a particular centrality measure, which often precluded comparison of the manipulability of different measures.

In this paper, we take a more general approach, where we pose the question about *the theoretical underpinnings behind quantifying the manipulability of centrality measures.* To answer this question, we take an *axiomatic approach* and formulate the problem characterized by a network, an evader node, a centrality measure and a set of allowed actions. We then introduce seven axioms that we believe are reasonable requirements for a measure of manipulability. We then prove that there exists only a single measure that satisfies all of them. We call it the Average Minimal Actions Required (AMAR) measure as it is equal to the inverse of the minimum number of actions that must be taken to manipulate the position of the evader node in the ranking averaged over all networks. We then use AMAR to experimentally quantify the manipulability of the four most popular centrality measures—Degree, Closeness, Betweenness and Eigenvector.

2 PROBLEM OF MANIPULABILITY

We study the difficulty with which a node—the *evader*—can affect its centrality in the network by manipulating the network structure. We consider undirected graphs $G = (V, E)$ drawn at random from a graph distribution $\mathcal{G}$ on a defined set of nodes V.

The evader, node in v, can manipulate the graph through actions. Such an action can be either adding or removing edges. For a set of actions S, by $S(G)$ we denote graph G after performing actions in S. To allow, for the sake of generality, we do not focus on one set of allowed actions (e.g., removing edges of the evader), instead, we allow for the arbitrary action function $\mathcal{A}$ that for each graph returns the set of allowed actions. Examples of such action functions can be found at the end of section 3.

By manipulating a centrality of the evader we can understand either hiding (decreasing the ranking) or exposing (increasing the

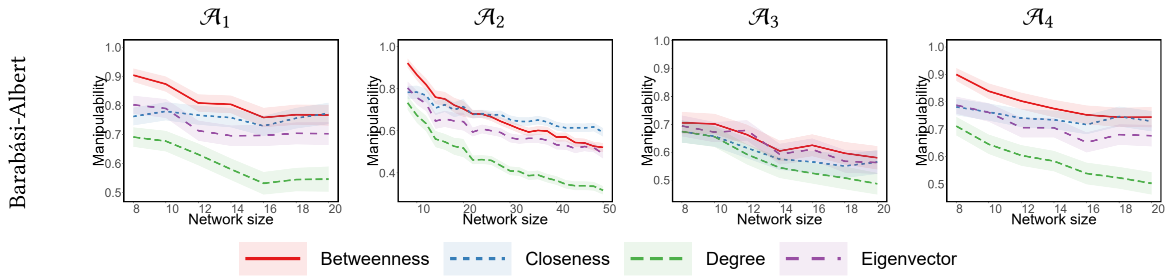

Figure 1: Fragment of the results of our experiments for one network model. Each column contains results for different set of actions. Values represent AMAR measure estimated using 400 networks. Colored areas represent 95% confidence intervals.

ranking). Here, we focus on hiding, however all results can be extended to exposing as well. For a given graph G_0, its node v, centrality measure F and set of actions A, the *impact set* is the collection of all subsets of A that hide v (i.e., decrease its ranking when performed). Formally, $I^F_{G_0,v}(A) = \{S \subseteq A : r^F_v(G_0) > r^F_v(S(G_0))\}$.

Finally, we define a *measure of manipulability* as a function, M, that for a every graph distribution $\mathcal{G}$ on space $\mathbb{G}^V$, node $v \in V$, centrality measure F, and action function $\mathcal{A}$ returns a real value from the interval $[0, 1]$. The greater the value, the easier it is for the evader to hide through manipulation.

3 AMAR MEASURE OF MANIPULABILITY

Definition of a measure of manipulability is very broad. To focus on more desirable measures of manipulability, we propose properties, i.e., axioms, that a measure of manipulability should satisfy.

- ***Unmanipulability:*** For every graph distribution $\mathcal{G}$, node v, centrality measure F, and action function $\mathcal{A}$, if it is certain that no combination of actions will hide an evader, i.e., $\mathbb{P}\big(I^F_{G,v}(\mathcal{A}(G)) = \emptyset\big) = 1$, then the manipulability is zero, i.e., $M(\mathcal{G}, v, F, \mathcal{A}) = 0$.
- ***Full Manipulability:*** For every graph distribution $\mathcal{G}$, node v, centrality measure F, and action function $\mathcal{A}$, it is sure to hide by any nonempty set of possible actions, i.e., $\mathbb{P}\big(I^F_{G,v}(\mathcal{A}) = \{S \subseteq \mathcal{A}(G) : S \neq \emptyset\}\big) = 1$, then $M(\mathcal{G}, v, F, \mathcal{A}) = 1$.
- ***Weak Dominance:*** For every graph distribution $\mathcal{G}$, node v, centrality measures F and F', and action functions $\mathcal{A}$ and $\mathcal{A}'$, if F and $\mathcal{A}$ *dominate* F' and $\mathcal{A}'$ (whenever the evader is hidden in F she is also hidden in F'), i.e., $\mathbb{P}\big(I^F_{G,v}(\mathcal{A}) = \{S \subseteq \mathcal{A}(G) : S \neq \emptyset\}\big) = 1$, then F and $\mathcal{A}$ is less manipulable than F' and $\mathcal{A}'$, i.e., $M(\mathcal{G}, v, F, \mathcal{A}) \leq M(\mathcal{G}, v, F', \mathcal{A}')$.
- ***Neutrality:*** A measure of manipulability should not unreasonably prefer one graph, node or centrality measure over the other, i.e., for every node v, bijections $f : V \to V$ and $g : \mathbb{G}^V \to \mathbb{G}^V$ centrality measures F and F', and action function $\mathcal{A}$ if $I^F_{G,v}(\mathcal{A}) = I^{F'}_{g(G),f(v)}(\mathcal{A})$ for every $G \in \mathbb{G}^V$ then $M(\mathcal{G}, v, F, \mathcal{A}) = M(\mathcal{G}', f(v), F', \mathcal{A})$, for every graph distributions $\mathcal{G}$ and $\mathcal{G}'$ such that $\mathbb{P}_{\mathcal{G}}(G = g(G_0)) = \mathbb{P}_{\mathcal{G}'}(G = G_0)$ for every G_0.
- ***Redundant Action:*** For every graph distribution $\mathcal{G}$, node v, centrality measures F, and action function $\mathcal{A}$, if there exist an action, which with portability 1 is redundant (if a set of actions containing a hides the evader, then this set with action a removed or exchanged for another action hides the evader as well), then $M(\mathcal{G}, v, F, \mathcal{A}) = M(\mathcal{G}, v, F, \mathcal{A} - a)$, where $(\mathcal{A} - a)(G) = \mathcal{A}(G) \setminus \{a\}$ for every G.
- ***Linearity:*** Manipulability over the combination of two network models is a combination of manipulabilities over these network models, i.e., For every two graph distributions $\mathcal{G}$ and $\mathcal{G}'$, node v, centrality measure F, action function $\mathcal{A}$, and two constants $x, y > 0$ such that $x + y = 1$ it holds that $M(x\mathcal{G} + y\mathcal{G}', v, F, \mathcal{A}) = xM(\mathcal{G}, v, F, \mathcal{A}) + yM(\mathcal{G}', v, F, \mathcal{A})$.

Our main result is that there exist only one measure of manipulability that satisfies all of our axioms. We will call it *Average Minimal Actions Required (AMAR)*. It is defined it as the average inverse of the minimal number of actions required to hide the evader. The cases in which it is impossible to hide the evader are counted as 0.

Formally, we first define *Minimal Actions Required (MAR)* function:

$$MAR(G, v, F, A) = \begin{cases} 0 & \text{if } I^F_{G,v}(A) = \emptyset, \\ \frac{1}{\min_{S \in I^F_{G,v}(A} |S|} & \text{otherwise.} \end{cases}$$

Then, building upon MAR function, we define Average Minimal Actions Required (AMAR) as its expected value over graph distribution, i.e., $AMAR(\mathcal{G}, v, F, \mathcal{A}) = \mathbb{E}\big(MAR(G, v, F, \mathcal{A}(G))\big)$. Our main technical result can be summarised by the following theorem

Theorem 3.1. *If a measure of manipulability, M, satisfies Unmanipulability, Full Manipulability, Weak Dominance, Neutrality, Redundant Action, and Linearity, then there exists non-decreasing function $f : [0, 1] \to [0, 1]$ such that $f(0) = 0$, $f(1) = 1$, such that for every graph distribution $\mathcal{G}$, node v, centrality measure F and action function $\mathcal{A}$ it holds that $M(\mathcal{G}, v, F, \mathcal{A}) = \mathbb{E}(f(MAR(G, v, F, \mathcal{A}(G)))$. If M satisfies additional axiom:* Normalisation, *then f is an identity and $M = AMAR$.*

Finally, we estimate the values of *AMAR* measure for different network models (here, only results for Barabási-Albert model [2] are presented), four centrality measures: *Degree* [23], *Closeness* [3], *Betweenness* [13], and *Eigenvector centrality* [5]; and four action functions: *All possible changes*: $\mathcal{A}_1(G) = \{a \subseteq V : |a| = 2\}$, *Removing evader's edges*: $\mathcal{A}_2(G) = \{a \in E[G] : v \in a\}$, *Adding edges between neighbors*: $\mathcal{A}_3(G) = \{a \subseteq N_G(v) : |a| = 2 \wedge a \notin E[G]\}$, *Local changes*: $\mathcal{A}_4(G) = \mathcal{A}_2(G) \cup \mathcal{A}_3(G)$. The evader in each network is set as a node with the top average ranking according to all four centrality measures. Results are depicted on the Figure 1.

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